

NCERT EXEMPLAR SELECTED PROBLEMS :

PROBLEM 9

Q26. Find the value of the expression $\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8}$

Sol. $\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8}$

$$\begin{aligned} &= \cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \left(\pi - \frac{3\pi}{8} \right) + \cos^4 \left(\pi - \frac{\pi}{8} \right) \\ &= \cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{\pi}{8} \\ &= 2 \left[\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} \right] \\ &= 2 \left[\cos^4 \frac{\pi}{8} + \cos^4 \left(\frac{\pi}{2} - \frac{\pi}{8} \right) \right] \\ &= 2 \left[\cos^4 \frac{\pi}{8} + \sin^4 \frac{\pi}{8} \right] \\ &= 2 \left[\left(\cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8} \right)^2 - 2 \cos^2 \frac{\pi}{8} \cdot \sin^2 \frac{\pi}{8} \right] \\ &= 2 \left[1 - 2 \cos^2 \frac{\pi}{8} \cdot \sin^2 \frac{\pi}{8} \right] \\ &= 2 - \left(2 \sin \frac{\pi}{8} \cdot \cos \frac{\pi}{8} \right)^2 \\ &= 2 - \left(\sin \frac{2\pi}{8} \right)^2 = 2 - \left(\frac{1}{\sqrt{2}} \right)^2 = 2 - \frac{1}{2} = \frac{3}{2} \end{aligned}$$