NCERT EXEMPLAR SELECTED PROBLEMS : PROBLEM 8

Q24. If x = sec φ-tanφandy = cosec φ + cot φ then show that xy + x -y +1=0.

Sol. $x = \sec \phi - \tan \phi \implies x = \dfrac{1 - \sin \phi}{\cos \phi}$

$y = \operatorname{cosec} \phi + \cot \phi \implies y = \dfrac{1 + \cos \phi}{\sin \phi}$

$\implies xy + x - y = \dfrac{1 - \sin \phi}{\cos \phi} \dfrac{1 + \cos \phi}{\sin \phi} + \dfrac{1 - \sin \phi}{\cos \phi} - \dfrac{1 + \cos \phi}{\sin \phi}$

$= \dfrac{(1 - \sin \phi)(1 + \cos \phi) + (1 - \sin \phi)\sin \phi - \cos \phi(1 + \cos \phi)}{\sin \phi \cos \phi}$

$= \dfrac{1 - \sin \phi + \cos \phi - \sin \phi \cos \phi + \sin \phi - \sin^2 \phi - \cos \phi - \cos^2 \phi}{\sin \phi \cos \phi}$

$= \dfrac{1 - \sin \phi \cos \phi - (\sin^2 \phi + \cos^2 \phi)}{\sin \phi \cos \phi} = -1$

$\therefore \quad xy + x - y - 1 = 0$