

NCERT EXEMPLAR SELECTED PROBLEMS :
PROBLEM 7

21. If $\cos(\theta + \phi) = m \cos(\theta - \phi)$, then prove that $\tan \theta = \frac{1-m}{1+m} \cot \phi$.

Sol. Given that, $\cos(\theta + \phi) = m \cos(\theta - \phi)$

$$\Rightarrow \frac{\cos(\theta + \phi)}{\cos(\theta - \phi)} = \frac{m}{1}$$

Using componendo and dividendo rule, we get

$$\frac{\cos(\theta - \phi) - \cos(\theta + \phi)}{\cos(\theta - \phi) + \cos(\theta + \phi)} = \frac{1-m}{1+m}$$

$$\Rightarrow \frac{2 \sin\left(\frac{\theta + \phi - \theta + \phi}{2}\right) \cdot \sin\left(\frac{\theta - \phi + \theta + \phi}{2}\right)}{2 \cos\left(\frac{\theta - \phi + \theta + \phi}{2}\right) \cdot \cos\left(\frac{\theta - \phi - \theta - \phi}{2}\right)} = \frac{1-m}{1+m}$$

$$\Rightarrow \frac{\sin \theta \cdot \sin \phi}{\cos \theta \cdot \cos \phi} = \frac{1-m}{1+m}$$

$$\Rightarrow \tan \theta = \left(\frac{1-m}{1+m}\right) \cot \phi$$