

NCERT EXEMPLAR SELECTED PROBLEMS :
PROBLEM 6

14. If $\tan \theta = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$, then show that $\sin \alpha + \cos \alpha = \sqrt{2} \cos \theta$.

Sol. We have, $\tan \theta = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$

$$\Rightarrow 1 + \tan^2 \theta = 1 + \frac{(\sin \alpha - \cos \alpha)^2}{(\sin \alpha + \cos \alpha)^2}$$

$$\Rightarrow \sec^2 \theta = \frac{(\sin^2 \alpha + \cos^2 \alpha + 2 \sin \alpha \cos \alpha + \sin^2 \alpha + \cos^2 \alpha - 2 \sin \alpha \cos \alpha)}{(\sin \alpha + \cos \alpha)^2}$$

$$\Rightarrow \sec^2 \theta = \frac{2}{(\sin \alpha + \cos \alpha)^2}$$

$$\Rightarrow (\sin \alpha + \cos \alpha)^2 = 2 \cos^2 \theta \Rightarrow \sin \alpha + \cos \alpha = \sqrt{2} \cos \theta$$