

NCERT EXEMPLAR SELECTED PROBLEMS :
PROBLEM 5

13. If $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$, then show that $\frac{\tan x}{\tan y} = \frac{a}{b}$.

Sol. Given that, $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$

Using componendo and dividendo, we get

$$\Rightarrow \frac{\sin(x+y) + \sin(x-y)}{\sin(x+y) - \sin(x-y)} = \frac{a+b+a-b}{a+b-a+b}$$

$$\Rightarrow \frac{2 \sin\left(\frac{x+y+x-y}{2}\right) \cdot \cos\left(\frac{x+y-x+y}{2}\right)}{2 \cos\left(\frac{x+y+x-y}{2}\right) \cdot \sin\left(\frac{x+y-x+y}{2}\right)} = \frac{2a}{2b}$$

$$\Rightarrow \frac{\sin x \cdot \cos y}{\cos x \cdot \sin y} = \frac{a}{b} \Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b}$$