PREVIOUS YEAR JEE EXAMPLES:

Question 1: ∫{[sin8x – cos8x] / [1 – 2 sin2x cos2x]} dx = __________.
Solution:
∫{[sin8x – cos8x] / [1 – 2 sin2x cos2x]} dx
= ∫{[(sin4x + cos4x) * (sin4x – cos4x)] / [(sin2x + cos2x)2 – 2 sin2x cos2x]} dx
= ∫(sin4x – cos4x) dx
= ∫[sin2x + cos2x] * [sin2x – cos2x] dx
= ∫(sin2x + cos2x) dx
= ∫–cos2xdx
= [–sin2x / 2] + c

Question 2: ∫x2dx / (a + bx)2 = ____________.
Solution:
Put a + bx = t
⇒ x = [t – a] / [b] and dx = dt / [b]
I =∫([t – a] / b)2 * [1 / t2] * [dt / b]
= [1 / b2]∫(1 – (2a / t) + [a2 * t–2]) dt
= [1 / b2] * [(t – 2a log t) – (a2 / t)]
= [1 / b2] [(x + a / b) – [2a / b] * log (a + bx) – [a2 / b] * [1 / (a + bx)]

Question 3: ∫[x5 / √(1 + x3)] dx = _________.
Solution:
Put 1 + x3 = t2
⇒ 3x2 dx = 2tdt and x3 = t2 – 1
So, ∫[x5 / √(1 + x3)] dx = ∫{[x2 * x3] / √(1 + x3)} dx
= [2 / 3] ∫{[(t2 – 1) * t] dt / [t]}
= [2 / 3] ∫(t2 – 1) dt
= [2 / 3] [(t3 / 3) – t] + c
= [2 / 3] [{(1 + x3)3/2 / 3} – {(1 + x3)½}]+ c

Question 4: ∫tan32x sec2x dx = ___________.
Solution:
∫tan32x sec2x dx = ∫[(sec2 2x – 1) sec2x * tan2x] dx
=∫sec32x tan2x dx –∫sec2x tan2x dx ……. (i)
Now, we take ∫sec32x tan2x dx
Put sec 2x = t
⇒ sec 2x tan 2x = dt/2, then it reduces to
[1 / 2] ∫t2 dt = t3 / 6

= [sec32x] / [6]
From (i), ∫sec32x tan2x dx –∫sec2x tan2x dx

= [sec3 2x / 6] – [sec2x / 2] + c
Trick: Let sec 2x = t, then sec 2x tan 2x dx = [1 / 2] dt
[1 / 2] ∫(t2 – 1) dt = [1 / 6]t3 – [1 / 2]t + c

= [sec3 2x / 6] – [sec 2x / 2] + c

Question 5: If ∫{[4ex + 6e–x] / [9ex – 4e–x]} dx = Ax + B log (9e2x – 4) + C, then find A, B and C.
Solution:
I =∫{[4ex + 6e–x] / [9ex – 4e–x]} dx
= [4 / 9]∫[9e2xdx] / [9e2x – 4] + 6∫[dx] / [9e2x – 4]
∫dx / [9e2x – 4] = [1 / 8] log (9e2x – 4) – [1 / 4] log3 – [1 / 4] x + constant
I = [35 / 36] log (9e2x – 4) – [3 / 2] x – [3 / 2] log3 + constant
Comparing with the given integral, we get
A = –3 / 2, B = 35 / 36, C = [–3 / 2] log3 + constant

Question 6: ∫x cos2x dx = ______.
Solution:
x cos2x dx = [1 / 2] ∫x (1 + cos2x) dx
= [x2 / 4] + [1 / 2] [(x sin2x) / (2) –∫(sin2x / 2) dx] + c
= [x2 / 4] + (x sin2x / 4) + (cos2x / 8) + c

Question 7: ∫x / [1 + x4] dx = ________.
Solution:
Put t = x2 ⇒ dt = 2x dx, therefore,
∫x / [1 + x4] dx = [1 / 2] ∫1 / [1 + t2] dt
= [1 / 2] tan–1 t + c
= [1 / 2] tan–1 x2 + c

Question 8: ∫[1 + x2] / √[1 – x2] dx = ________.
Solution:
Put x = sinθ ⇒ dx = cosθ dθ, then it reduces to
∫(1 + sin2θ) dθ = θ + [1 / 2]∫(1 – cos2θ) dθ
= [3θ / 2] – [1 / 2] sinθ * √[1 – sin2θ] + c
= [3 / 2] sin–1 x – [1 / 2]x √[1 – x2] + c

Question 9: ∫√(1 + sin [x / 2]) dx = _________.
Solution:
∫√(1 + sin [x / 2]) dx = ∫√(sin2 [x / 4] + cos2 [x / 4] + 2 sin [x / 4] cos [x / 4]) dx
= ∫(sin [x / 4] + cos [x / 4]) dx
= 4 (sin [x / 4] – cos [x / 4]) + c

Question 10: ∫[sinx] / [sin (x – α)] dx = ________.
Solution:
∫[sinx] / [sin (x – α)] dx =
∫[sin (x – α + α)] / [sin (x – α)] dx
= ∫{[(sin (x – α) cosα + cos (x – α) sinα] / [sin (x – α)]} dx
= ∫cosα dx +∫sinα * cot (x – α) dx
= x cosα + sinα * log sin (x – α) + c

Question 11: ∫(log x)2 dx = _______.
Solution:
∫(log x)2 dx
Put log x = t
⇒ et = x
⇒ dx = et dt, then it reduces to
∫t2 * et dt = t2 * et – 2t * et + 2et + c
= x (log x)2 – 2x log x + 2x + c

Question 12: ∫[cos2θ] * log ([cosθ + sinθ] / [cosθ – sinθ]) dθ = ___________.
Solution:
We know that
log ([cosθ + sinθ] / [cosθ – sinθ]) = log ([1 + tanθ] / [1 – tanθ]) = log tan (π / 4 + θ) ∫secθ dθ = log tan (π / 4 + θ / 2)
∫sec2θ dθ = [1 / 2] * log tan (π / 4 + θ)
2 sec2θ = [d / dθ] log tan (π / 4 + θ) …….(i)
Integrating the given expression by parts, we get
I = [1 / 2] sin2θ log tan (π / 4 + θ) – [1 / 2] ∫[sin2θ * 2 sec2θ] dθ by (i)
= [1 / 2] sin2θ log tan (π / 4 + θ) –∫tan2θ dθ
= [1 / 2] sin2θ log tan (π / 4 + θ) – [1 / 2] log sec2θ

Question 13: ∫dx / (sinx + sin2x) = _________.
Solution:
I = ∫dx / [sinx (1 + 2cosx)]
= ∫[sinx dx] / [sin2x * (1 + 2 cosx)]
= ∫sinx dx / {(1 – cosx) * (1 + cosx) * (1 + 2 cosx)}
Now differential coefficient of cosx is – sinx, which is given in numerator and hence, we make the substitution cosx = t ⇒ –sinx dx = dt
I = –∫dt / [(1 – t) (1 + t) (1 + 2t)]
We split the integrand into partial fractions
I = –∫[{1 / 6 (1 – t)} – {1 / 2 (1 + t)} + {4 / 3 (1 + 2t)}] dt
[1 / 6] log (1 – cosx) + [1 / 2] log (1 + cosx) – [2 / 3] log (1 + 2 cosx)

Question 14: For which of the following values of m, the area of the region bounded by the curve y = x − x2 and the line y = mx equals 9 / 2?

Solution:

The equation of curve is y = x − x2

⇒ x2 − x = −y

(x − [1 / 2])2 = − (y − [1 / 4])

This is a parabola whose vertex is (12, 14)

Hence, point of intersection of the curve and the line x − x2 = mx

⇒ x (1 − x − m) = 0 i.e., x =0 or x = 1 − m

[9 / 2] = ∫1−m0 (x − x2 − mx) dx

= ([x2 / 2] − [x3 / 3] − [mx2 / 2])1−m0

= [(1 − m)] * [(1 − m)2 / 2] − [(1 − m)3 / 3] = [(1 − m)3/ 6]

(1 − m)3 = [6 * 9] / [2] = 27

⇒ 1 − m = 271/3 = 3

⇒ m = −2

Also, (1 − m)3 − 33 = 0

(1 − m −3) [(1 − m)2 + 9 + (1 − m)3] = 0

(1 − m)2 + 3 (1 − m) + 9 = 0

m2 − 2m + 1 − 3m + 3 + 9 = 0

m2 − 5m + 13 = 0

m = (5 ± √[25 − 52]) / 2 i.e., m is imaginary

Hence, m = −2.

Question 15: Let f : R → R and g : R → R be continuous functions, then the value of the integral ∫π/2−π/2 [f (x) + f (−x)] [g (x) − g (−x)] dx = ____________.

Solution:

Let h(x) = {f (x) + f (−x)} {g (x) − g (−x)}

h (−x) = {f (−x) + f (x)} {g (−x) − g (x)}

= −{f (−x) + f (x)} {g (x) − g (−x)}

= − h (x)

Therefore, ∫π/2−π/2 h (x) dx = 0.