

Lecture related problems with solutions

Q1) Find the length of the tangent from $(12, -9)$ to the circle

$$3x^2 + 3y^2 - 7x + 22y + 9 = 0$$

Ans) Dividing the equation of the circle by 3, we get the standard form

$$x^2 + y^2 - 7/3x + 22/3y + 3 = 0$$

The required length of the tangent from $(12, -9)$ is

$$\sqrt{(12^2 + (-9)^2 - 7/3(12) + 22/3(-9) + 3)} = \sqrt{134}$$

Q2) The tangent to circle $x^2 + y^2 = 5$ at $(1, -2)$ also touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$

Find the coordinates of the corresponding point of contact.

Ans) equation of tangent from $(1, -2)$ to $x^2 + y^2 = 5$ is $x - 2y - 5 = 0$.

Now putting $x = 2y + 5$ in second circle we get $y^2 + 2y + 1 = 0$, $y = -1, x = 3$.

Thus, the point of contact is $(3, -1)$.

Q3) Find the angle between the two tangents from the origin to the circle

$$(x-7)^2 + (y+1)^2 = 25.$$

Ans) Any line through $(0, 0)$ be $y = mx$ and it's a tangent to circle $(x-7)^2 + (y+1)^2 = 25$

$$\frac{|-1-7m|}{\sqrt{1+m^2}} = 5 \Rightarrow m = 3/4, -4/3.$$

The product of both is -1 so they are perpendicular to each other

Hence, the angle between the two tangents is $\pi/2$.

Q4) Find the equation of normal to the circle $2x^2 + 2y^2 - 2x - 5y + 3 = 0$ at $(1, 1)$

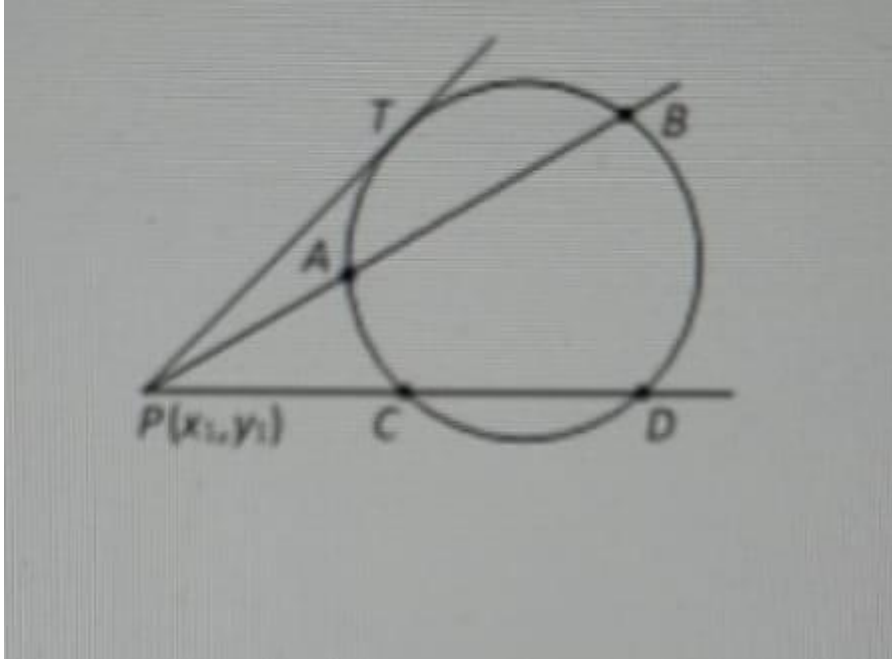
Ans) By solving we get the centre of the circle is $(1/2, 5/4)$

Normal to circle at point $(1, 1)$ is line passing through the points $(1, 1)$ and $(1/2, 5/4)$

Which is $x + 2y = 3$.

Q5) Let $P(x_1, y_1)$ be a point outside the circle PAB and PCD drawn two secants. The power of

$P(x_1, y_1)$ with respect to $s = x^2 + y^2 + 2gx + 2fy + c = 0$ is equal to $PA \cdot PB$ which is



Ans) $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = S_1$

$\therefore PA \cdot PB = (S_1) = \text{Square of the length of tangent.}$