Lecture related problems with solutions

1) Find the length of the x-intercept and y-intercept made by the circle $x^2 + y^2 - 4x - 6y - 5 = 0$ with the co-ordinate axes.

Sol) Given equation of the circle is $x^2 + y^2 - 4x - 6y - 5 = 0$.

Now comparing the given equation with the general equation of the circle, we get $g = -2$ and $f = -3$ and $c = -5$.

So, length of the x-intercept $= 2(g^2-c)^{1/2} = 2(4-(-5))^{1/2}=2*3=6$.

length of the y-intercept $= 2(f^2-c)^{1/2} = 2(9-(-5))^{1/2}=2(14)^{1/2}$.

2) Find the equation of a circle which touches the y-axis at a distance -3

From the origin and cuts an intercept of 8 units with the positive direction of X-axis.

Sol) Let the equation of the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$ .....(1)

According to the question the equation (1) touches the Y-axis so

$C=f^2$ ....(2)

the point (0, -3) lies on the circle (1) so by putting the values in (1)

we get $9 - 6f + c = 0$ .....(3)

from (2) and (3) we get $9 - 6f + f^2 = 0 \Rightarrow f=3$.

Now put $f=3$ in (3) we get $c=9$.

Again, according to the problem the equation of the circle (1) cuts an intercept of 8 units with the positive direction of x-axis.

So $2(g^2-c)^{1/2}=8$

On solving we get **$g=+5, -5$.**

Hence, the required equation of the circle is

$x^2 + y^2 \pm 10x + 6y + 9 = 0$.

3) Find the points of intersection of the circle with the line $(x - 2)^2 + (y + 3)^2 = 4$ and $2x + 2y = -1$

Sol) $y = - x - 1/2$ now keep the y value in the circle equation we get

$(x - 2)^2 + (- x - 1/2 + 3)^2 = 4 \Rightarrow 2 x^2 - 9 x + 25/4 = 0$

On solving we get $x = (9 + \sqrt{31}) / 4$ and $(9 - \sqrt{31}) / 4$.

Now by solving we get $y = (-11 - \sqrt{31}) / 4$ and $y = (-11 + \sqrt{31}) / 4$.

So the point of intersections of circle and line are

$((9 + \sqrt{31}) / 4 , (-11 - \sqrt{31}) / 4)$ and $((9 - \sqrt{31}) / 4 , (-11 + \sqrt{31}) / 4)$.