Q12: The moment of inertia of a uniform cylinder of length and radius R about its perpendicular bisector is I. What is the ratio /R such that the moment of inertia is minimum?

(a) 1

(b) $3\sqrt{2}$

(c) $\sqrt{3}/2$

(d) $\sqrt{3/2}$

Solution

Moment of Inertia I = m [$l^2/12 + R^2/4$]

Volume = $\pi R^2 l$

Writing Moment of inertia in terms of volume, we get

I = m/4($V/\pi l + l^2/3$)

Differentiating the above equation we get

dI/d/ = m/4 ($-V/\pi l^2 + 2l/3$)

For maxima and minima, dI/d/ = 0

So m/4 ($-V/\pi l^2 + 2l/3$) = 0

$V/\pi l^2 = 2l/3$

$R^2/l = 2l/3$ (Volume = $\pi R^2 l$)

$l^2/R^2 = 3/2$

$$l/R = \sqrt{\frac{3}{2}}$$

Answer: (d) $\sqrt{3/2}$