Integration of Trigonometric function

Case I :- $\int \frac{dx}{a+b\cos^2 x}$, $\int \frac{dx}{a+b\sin^2 x}$, $\int \frac{dx}{a\cos^2 x+b\sin^2 x}$, $\int \frac{dx}{a\cos^2 x+b\sin^2 x+c}$

divide Nu & Deno by $\cos^2 x$ or $\sin^2 x$

Case II :- $\int \frac{dx}{a+b\cos x}$, $\int \frac{dx}{a+b\sin x}$, $\int \frac{dx}{a\cos x+b\sin x}$,

$\int \frac{dx}{a\cos x+b\sin x+c}$ put $\cos x = \frac{1-\tan^2 \frac{x}{2}}{1+\tan^2 \frac{x}{2}}$

$\sin x = \frac{2\tan \frac{x}{2}}{1+\tan^2 \frac{x}{2}}$

Case III :- $\int \sin^n x \cdot \cos^m x$ $(m, n \in N)$

(i) If m, n are odd then $\sin x = t$ or $\cos x = t$
(ii) If m even, n odd then $\cos x = t$
(iii) If m odd, n even the put $\sin x = t$
(iv) If m, n are even then use trigonometric identities

Ex – $\int (\sin x)^4 (\cos x)^5 . dx$

$= \int \sin^4 x \cdot \cos^4 x \cdot \cos x . dx = \int \sin^4 x (1-\sin^2 x)^2 \cos x . dx$

$= \int t^4 (1-t^2)^2 . dt$

Ex $\int \sin^4 x \cos^6 x \, dx = \int \left(\frac{\sin 2x}{2}\right)^2 \times \left(1 + \frac{\cos 2x}{2}\right) dx$

$= \frac{1}{2^5} \int (\sin 2x \cdot \cos 2x + \sin^3 2x) \, dx$

$= \frac{1}{2^9} \int 2 \sin 4x (3 \sin 2x - \sin 6x) \, dx$

then use $2 \sin A \sin B$

Case-IV $\int \frac{(a \sin x + b \cos x + c)}{d \sin x + e \cos x + f} dx$

put $a \sin x + b \cos x + c = A(d \sin x + e \cos x + f)$
$+ B(\text{diff of } (d \sin x + e \cos x + f)) + C$

Case V. $\int \frac{(a \sin x + b \cos x)}{d \sin x + e \cos x} dx$

put $a \sin x + b \cos x = A(d \sin x + e \cos x) + B(\text{diff of}$
$(d \sin x + e \cos x))$

Ex - $\int \frac{\sin x + 2\cos x + 3}{4 \sin x + 5 \cos x + 6} dx$

$\sin x + 2\cos x + 3 = A(4 \sin x + 5 \cos x + 6) + B(4 \cos x - 5 \sin x) + C$

$4A - 5B = 1 \rightarrow 16A - 20B = 4$ $\boxed{A = \frac{14}{41}}$

$5A + 4B = 2$ $\quad 25A + 20B = 10$

$6A + C = 3$

$$\boxed{\int\left[A\left(\frac{4\sin x+5\cos x+6}{\text{Deno}}\right)+B\left(\frac{4\cos x-5\sin x}{\text{Deno}}\right)+\frac{C}{\text{Deno}}\right].dx}$$

Case VI $\int\sqrt{a\sec^2x+b}$, $\int\sqrt{a\,\text{cosec}^2+b}\,.dx$, $\int\sqrt{a\tan^2x+b}$, $\int\sqrt{a\cot^2x+b}$

integrate by using rationalisation

Ex $\int\sqrt{a\sec^2x+b}\,.dx$

$$=\int\frac{a\sec^2x+b}{\sqrt{a\sec^2x+b}}=a\int\frac{\sec^2x\,.dx}{\sqrt{a+b+a\tan^2x}}+b\int\frac{b\,dx\cos x}{\sqrt{a+b\cos^2x}}$$

put $\tan x=t$ & $\sin x=p$

$$=a\int\frac{dt}{\sqrt{a+b+at^2}}+\int\frac{b\,dp}{\sqrt{a+b-bp^2}}$$

Case VII $\int f(\sin 2x)(\cos x-\sin x)\,dx$ or $\int f(\sin 2x)(\cos x+\sin x).dx$

here $\sin 2x=(\sin x+\cos x)^2-1$ $\quad\sin 2x=1-(\sin x-\cos x)^2$

Case VIII $\int\frac{b+a\sin x}{(a+b\sin x)^2}.dx$ divide Nu & Deno. by $\cos^2x$

$\int\frac{b+a\cos x}{(a+b\cos x)^2}$ divide Nu & Deno by $\sin^2x$

Case IX $\int \tan^m x \cdot \sec^n x \cdot dx$

n even, put $\tan x = t$

n odd, put $\sec x = t$

Eg - $\int \tan^2 x \cdot \sec^3 x \cdot dx$

$\Rightarrow \int \sec x \cdot \tan x \sqrt{\sec^2 x - 1} \cdot \sec^2 x \cdot dx$

$\sec x = t$

$\int t^2 \sqrt{t^2-1} \cdot dt \Rightarrow \int (t^2-1)^{\frac{3}{2}} - \int \sqrt{t^2-1} \cdot dt$

$= \int (t^2-1)^{\frac{3}{2}} \cdot dt = \int \tan^3\theta \cdot \sec\theta \cdot \tan\theta \cdot d\theta$

$= \int (\sec^5\theta + \sec\theta - 2\sec^3\theta) \cdot d\theta$

$\hookrightarrow 2\sqrt{1+\tan^2\theta} \cdot \sec^2\theta \cdot d\theta$

$I = \int \sec^3\theta \cdot \sec^2\theta \cdot d\theta$
(I) (II)

$t = \sec\theta$

$I = \sec^3\theta \cdot \tan\theta - \int 3\sec^2\theta \cdot \tan^2\theta$

$I = \sec^3\theta \cdot \tan\theta - 3\int \sec^3\theta(\sec^2\theta - 1) \cdot d\theta$

then taking $\int \sec^5\theta \cdot d\theta = I$ and solving.

$$\Rightarrow I = \int \underset{II}{1} \cdot \underset{I}{(t^2-1)^{\frac{3}{2}}} \cdot dt$$

$$I = (t^2-1)^{\frac{3}{2}} t - \frac{3}{2}\int (t^2-1)^{\frac{1}{2}} 2t \cdot t \cdot dt$$

$$\therefore I = t(t^2-1)^{\frac{3}{2}} - 3I - 3\int \sqrt{t^2-1} \cdot dt$$

$$4I = t(t^2-1)^{\frac{3}{2}} - 3\int \sqrt{t^2-1} \cdot dt$$

$$\therefore \int \tan^2 x \cdot \sec^2 x \cdot dx = I - \sqrt{t^2-1} \cdot dt$$

$$= \boxed{\frac{t(t^2-1)^{\frac{3}{2}}}{4} - \frac{7}{4}\int \sqrt{t^2-1} \cdot + C}$$

M-2 $\int \frac{(t^2-1)^2}{\sqrt{t^2-1}} \cdot dt = \int \frac{t^4 - 2t^2 + 1}{\sqrt{t^2-1}} = \int \frac{t^4}{\sqrt{t^2-1}} - \int \frac{2t^2 - 2 + 1}{\sqrt{t^2-1}}$

can be done

$\int \frac{t^4}{\sqrt{t^2-1}} \rightsquigarrow t = \sec\theta$

$$\int \frac{t^6 \cdot dt}{t^3\sqrt{1-\frac{1}{t^2}}} = \int \frac{\cancel{2p}\, dp}{p}\left(\frac{1}{1-p^2}\right)^3$$

M-3 $\int \tan^2 x \cdot \sec^3 x \cdot dx$ $\quad \tan^2 x = t$

$2\tan x \cdot \sec^2 x \cdot dx = dt$

$$\int \frac{t\sqrt{1+t}}{2\sqrt{t}} = \frac{1}{2}\int \sqrt{\left(t+\frac{1}{2}\right)^2 - \frac{1}{4}} \cdot dt$$

(1) $\int \sin 4x \cdot e^{\tan^2 x} \cdot dx$

$= 2\int \sin 2x \cdot \cos 2x \cdot e^{\tan^2 x} dx = 2\int \frac{2\tan x}{1+\tan^2 x} \cdot \frac{1-\tan^2 x}{1+\tan^2 x} \cdot \frac{\sec^2 x}{(1+\tan^2 x)}$

$\tan^2 x = t \qquad 2\tan x \sec^2 x \cdot dx = dt$

$\therefore \cancel{e^{\tan^2 x} = t \Rightarrow 2\tan x}$

$= 2\int \frac{1-t}{(1+t)^3} e^t \cdot dt$

$= 2\int \left(\frac{-1-t+2}{(1+t)^3}\right) e^t$

$= 2\int \left(\underbrace{\frac{-1}{(1+t)^2}}_{f(x)} + \underbrace{\frac{2}{(1+t)^3}}_{f'(x)}\right) e^t \cdot dt$

$= 2 \cdot \frac{-e^t}{(t+1)^2} = \boxed{\frac{-2e^t}{(t+1)^2} + c} \quad \boxed{t = \tan^2 x}$

Q. 2 $\int \frac{\sqrt{\cos 2x}}{\sin x} \cdot dx = -\ln\left|t + \sqrt{t^2 - 1}\right| + \sqrt{2} \ln\left|p + \sqrt{p^2 - \frac{1}{2}}\right| + c$

$t = \cot x \qquad p = \cos x$

$= \int \frac{\sqrt{\cos 2x}}{\sin x} = \int \sqrt{\operatorname{cosec}^2 x - 2} \cdot dx = \int \frac{\operatorname{cosec}^2 x - 2}{\sqrt{\operatorname{cosec}^2 x - 2}} \cdot dx = \frac{\operatorname{cosec}^2 x}{\sqrt{\cot^2 x - 1}} - \frac{\sqrt{2}\sin x}{\sqrt{\cos^2 x - \frac{1}{2}}}$

put $t = \cot x$ $\qquad$ put $p = \cos x$

$= \int \frac{-dt}{\sqrt{t^2 - 1}} + \int \frac{\sqrt{2}\, dp}{\sqrt{t^2 - \frac{1}{2}}} = \boxed{-\ln\left|\cot x + \sqrt{\cot^2 x - 1}\right| + \sqrt{2} \ln\left|\cos x + \sqrt{\cos^2 x - \frac{1}{2}}\right| + c}$

(3) $\int \frac{5\tan x}{\tan x - 2}\,dx \Rightarrow x + 2\ln(\sin x - 2\cos x) + C$

$$= \frac{5\sin x}{\sin x - 2\cos x} = 5\left[\frac{A(\sin x - 2\cos x) + B(2\sin x + \cos x)}{\sin x - 2\cos x}\right]$$

$\therefore A + 2B = 1$ & $B = 2A$ $\therefore A = \frac{1}{5}$ & $B = \frac{2}{5}$

$$\therefore \int \frac{5\sin x}{\sin x - 2\cos x} = \int \left(1 + \frac{2(2\sin x + \cos x)}{\sin x - 2\cos x}\right).dx$$

$$= \boxed{x + 2\ln(\sin x - 2\cos x) + c}$$

(4) $\int \operatorname{cosec}^5 x\,dx \longrightarrow \boxed{-\frac{1}{4}\operatorname{cosec}^3 x \cot x - \frac{3}{4}\left(\frac{t}{2}\sqrt{1+t^2} + \frac{1}{2}\ln\left|t + \sqrt{1+t^2}\right|\right)}$

$\operatorname{cosec} x = t$ $\qquad -\operatorname{cosec} x \cot x\,dx = dt$ $\qquad t = \cot x$

$$\therefore \int \operatorname{cosec}^5 x\,dx = \int -\frac{t^4}{\sqrt{t^2-1}}.dt$$

$$= -\left(\int \frac{t^5}{\sqrt{t^2-1}} - \int t.\frac{(4t^3(t^2-1) - 2t.t^4)}{t^2-1}.dt\right) = \frac{-t^5}{\sqrt{t^2-1}} + \int \frac{2t^6 - 2t^4 - 2t^4 + 2t^2 - 2t^2 + 2 - 2}{t^2-1}$$

$$= \boxed{\frac{-t^5}{\sqrt{t^2-1}} + \frac{2}{5}t^5 - \frac{2}{3}t^3 - 2t + \ln\frac{t+1}{t-1} + c} \qquad \boxed{t = \operatorname{cosec} x}$$

(5) $\int \sin^2 x \cdot \cos^4 x\,dx$

$$= \int \frac{1}{16}(1 - \cos 4x)(\cos 2x + 1).dx = \int \frac{1}{16}(\cos 2x - \cos 4x\cos 2x - \cos 4x + 1)$$

$$= \int \frac{1}{16}\left(\cos 2x - \frac{\cos 6x}{2} - \frac{\cos 2x}{2} - \cos 4x + 1\right)$$

$$= \frac{1}{16}\left(\frac{\sin 2x}{2} - \frac{\sin 6x}{12} - \frac{\sin 2x}{4} - \frac{\sin 4x}{4} + x\right) + C$$

(6) $\int \sin^3 x \,(\cos x)^{\frac{1}{3}} \cdot dx$

(7) $\int \sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2+8x+13}}\right) \cdot dx$

(8) $\int \frac{dx}{1+\sqrt{x^2+2x+2}}$

Ans 6 $\therefore \cos x = t^3$ $\quad\therefore -\sin x \cdot dx = 3t^2 \cdot dt$

$$= -\int -(1-t^6)\, t \cdot 3t^2 \cdot dt$$

$$= \int (3t^9 - 3t^3) \cdot dt$$

$$= \boxed{\frac{3}{10}t^{10} - \frac{3}{4}t^4 + C} \qquad \boxed{t = (\cos x)^{\frac{1}{3}}}$$

Ans 7 $\frac{2x+2}{\sqrt{(2x+2)^2+9}} = \sin t$, $(x+1) = \frac{3}{2}\tan\theta$

$dx = \frac{3}{2}\sec^2\theta \cdot d\theta$

$$= \int \sin^{-1}\sin\theta \cdot \frac{3}{2}\sec^2\theta \cdot d\theta$$

$$= \int \theta \cdot \frac{3}{2}\sec^2\theta \cdot d\theta = \frac{3}{2}\left(\theta \cdot \tan\theta - \int \tan\theta \cdot d\theta\right)$$

$$= \boxed{\frac{3}{2}\left(\theta\tan\theta + \ln\cos\theta\right) + C} \qquad \boxed{\theta = \tan^{-1}\frac{2}{3}(x+1)}$$

Ans 8 $\displaystyle\int \frac{dx}{1+\sqrt{(x+1)^2+1}} = \int \frac{\sqrt{(x+1)^2+1}-1}{(x+1)^2}\, dx$

$\because$ put $(x+1) = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$$= \int \frac{(\sec\theta - 1)\sec^2\theta \cdot d\theta}{\tan^2\theta}$$

$$= \int \frac{(\sec\theta - 1)}{\sin^2\theta} \cdot d\theta = \int \frac{d\theta}{\sin^2\theta\cos\theta} + \cot\theta$$

put $\sin\theta = p$

$\cancel{\cos\theta} \cdot d\theta = \dfrac{dp}{\cos\theta}$

$$= \int \frac{dp}{p^2(1-p^2)}$$

$$= \int \frac{dp}{p^2} + \frac{dp}{1-p^2} = \cancel{-\frac{1}{p} + \int \frac{1}{2}\left(\frac{1}{1-p} + \frac{1}{1+p}\right) \cdot dp} = -\frac{1}{p} + \frac{1}{2}\ln\frac{1+p}{1-p}$$

$$= \cancel{\boxed{-\frac{1}{p} - \frac{1}{2}\ln\frac{p-1}{p+1}}} = \cancel{-\frac{1}{p} + \frac{1}{2}\ln(p+1)}$$

Net integration

$$= \int \frac{dp}{p^2(1-p^2)} + \cot\theta$$

$$= \cancel{\boxed{-\frac{1}{p} + \frac{1}{2}\ln\frac{1-p^2}{1+p} + \cot\theta + C}} \quad \cancel{\boxed{x+1 = \tan\theta \quad \sin\theta = p}}$$

$$= \cancel{-\csc\theta - \frac{1}{2}\ln\frac{\sin\theta - 1}{\sin\theta + 1}}$$

$$= \boxed{-\frac{1}{p} + \frac{1}{2}\ln\frac{1+p}{1-p} + \cot\theta + C} \quad \boxed{x+1 = \tan\theta \quad \sin\theta = p}$$

$$\boxed{-\csc\theta + \frac{1}{2}\ln\frac{1+\sin\theta}{1-\sin\theta} + \cot\theta + C}$$ where $\boxed{\tan\theta = x+1}$

① $\displaystyle\int \frac{x^2+20 \cdot dx}{(x\sin x+5\cos x)^2} \Rightarrow -\cot\left(x+\tan^{-1}\frac{5}{x}\right)+c$

$$\cancel{\int \frac{dx}{\left(\frac{x\sin x}{\sqrt{x^2+25}}+\frac{5\cos x}{\sqrt{x^2+25}}\right)^2} = \int \frac{dx}{\sin^2\left(x+\sin^{-1}\frac{5}{\sqrt{x^2+25}}\right)}}$$

$$\cancel{=\int \operatorname{cosec}^2\left(x+\sin^{-1}\frac{5}{\sqrt{x^2+25}}\right) \cdot dx} \quad \frac{5 \cdot dx^2}{(x\sin x+5\cos x)^2}$$

$$= \int \frac{(x^2+20)\cdot dx}{(x^2+25)\left(\sin^2\left(x+\tan^{-1}\frac{5}{x}\right)\right)} \qquad \therefore x+\tan^{-1}\frac{5}{x} = t$$

$$= \int \operatorname{cosec}^2 t \cdot dt. \qquad 1+\frac{-\frac{5}{x^2}}{1+\left(\frac{5}{x}\right)^2} = dt$$

$$= \boxed{-\cot\left(x+\tan^{-1}\frac{5}{x}\right)+c} \qquad = \left(\frac{x^2+20}{x^2+25}\right)\cdot dx = dt$$

②) $\sqrt{\sin 2x} \cdot \cos x \cdot dx$

$$\frac{1}{2}\int \sqrt{\sin 2x}\,(\cos x+\sin x) \cdot dx + \sqrt{\sin 2x}\,(\cos x-\sin x) \cdot dx$$

$$= \frac{1}{2}\int \sqrt{1-(\sin x-\cos x)^2}\,(\cos x+\sin x) \cdot dx - \sqrt{(\sin x+\cos x)^2-1}\,(\cos x-\sin x) \cdot dx$$

$$= \frac{1}{2}\int \sqrt{1-t^2} \cdot dt + \frac{1}{2}\int \sqrt{p^2 [redacted]} \cdot dp$$

$$= \frac{1}{2}\left(\frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}\sin^{-1} x\right) + \frac{1}{2}\left(\frac{p}{2}\sqrt{p^2-1} - \frac{1}{2}\ln\left(p+\sqrt{p^2-1}\right)\right)+c$$

$\boxed{t = \sin x-\cos x}$ $\quad$ $\boxed{p = \sin x+\cos x}$

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(3) $\dfrac{e^{2x}-e^{x}+1}{(e^{x}\sin x+\cos x)(-e^{x}\cos x+\sin x)}$

$= \int \dfrac{(e^{2x}-e^{x}+1)\,dx}{(e^{2x}+1)-\sin(x+\theta)\cos(x+\theta)}$ $\qquad \theta = \tan^{-1} e^{-x}$

$\int \dfrac{-2(e^{2x}-e^{x}+1)}{(e^{2x}+1)\cdot\sin(2x+2\theta)}$ $\qquad 2x+2\theta = t$

$\qquad 2 - \dfrac{2e^{x}}{1+e^{2x}} = dt$

$= \int -\operatorname{cosec} t\,.dt$ $\qquad 2\left(\dfrac{e^{2x}+1-e^{x}}{e^{2x}+1}\right) = dt$

$= \boxed{\ln(\operatorname{cosec} t+\cot t)+c}$ $\therefore$ $\boxed{t = 2x+2\tan^{-1}e^{-x}}$

(4) $\int \dfrac{(\sin x)\,.\,dx}{\sin^{3}x+\cos^{3}x}$

$= \dfrac{\sqrt{2}\sin x}{\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)(2-\sin 2x)} = \dfrac{\sqrt{2}\sin x}{\sin\left(x+\frac{\pi}{4}\right)(2-\sin 2x)} = \int \dfrac{\sqrt{2}\sin\left(t-\frac{\pi}{4}\right)}{\sin t\,(2+\cos 2t)}$

$x+\dfrac{\pi}{4} = t$

$= \dfrac{\sqrt{2}\left(\frac{\sin t}{\sqrt{2}}-\frac{\cot t}{\sqrt{2}}\right)\sqrt{2}}{\sin t\,(2+\cos 2t)} = \dfrac{(\sin t-\cos t)\,.dt}{\sin t\,(2+\cos 2t)} = \int \dfrac{\sec^{2}t\,.dt}{(3+\tan^{2}t)} - \dfrac{\cot t\sec^{2}t\,.dt}{(3+\tan^{2}t)}$

$\therefore \tan t = p$
$\sec^{2}t\,.dt = dp$

$= \int \dfrac{dp}{3+p^{2}} - \dfrac{dp}{p(3+p^{2})} = \int \dfrac{dp}{3+p^{2}} - \dfrac{1}{3}\left(\dfrac{dp}{p} - \dfrac{p.\,dp}{3+p^{2}}\right)$

$= \cancel{\dfrac{1}{\sqrt{3}}\tan^{-1}\dfrac{\tan(x+\frac{\pi}{4})}{\sqrt{3}}}$ $= \boxed{\dfrac{1}{\sqrt{3}}\tan^{-1}\left(\dfrac{\tan(x+\frac{\pi}{4})}{\sqrt{3}}\right) - \dfrac{1}{3}\ln\tan\left(x+\dfrac{\pi}{4}\right) + \dfrac{1}{6}\ln\left(3+\tan^{2}\left(x+\dfrac{\pi}{4}\right)\right) + c}$

(5) $\int (\cos 2x)^{\frac{3}{2}}\cos x\,.dx$

(6) $\int \dfrac{\sin^{2}x\,.\,dx}{e^{2mx}(\cos x - m\sin x)^{2}}$

(7) $\int e^{x\sin x+\cos x}\left(\dfrac{x^{4}\cos^{3}x - x\sin x+\cos x}{x^{2}\cos^{2}x}\right).dx$

⑧ $\int \frac{\operatorname{cosec}^2 x \cdot dx}{(\operatorname{cosec} x + \cot x)^{\frac{9}{2}}}$

Ans 5 $\int (\cos 2x)^{\frac{3}{2}} \cos x \cdot dx = \int (1-2\sin^2 x)^{\frac{3}{2}} \cdot \cos x \cdot dx$

$\sin x = t \qquad \cos x \cdot dx = dt$

$= \int (1-2t^2)^{\frac{3}{2}} \cdot dt = t(1-2t^2)^{\frac{3}{2}} - \frac{3}{2} \int t \cdot (1-2t^2)^{\frac{1}{2}} \cdot dt$

$1-2t^2 = p^2 \qquad \therefore -\cancel{2}t \cdot dt = \cancel{2}p \cdot dp$

$= t(1-2t^2)^{\frac{3}{2}} + \frac{3}{4} \int p^2 \cdot dp = \boxed{t(1-2t^2)^{\frac{3}{2}} + \frac{p^3}{4} + C}$

$\boxed{t(1-2t^2)^{\frac{3}{2}} + \frac{(1-2t^2)^{\frac{3}{2}}}{4} + C}$ $\boxed{t = \sin x}$

Ans 6 $e^{-2mx} \left(\frac{-2m(f(x))}{\cos x - m\sin x} + \frac{f'(x)(\cos x - m\sin x) + f(x)(m\cos x + \sin x)}{(\cos x - m\sin x)^2} \right)$

$\therefore \frac{e^{-2mx}}{(\cos x - m\sin x)^2} \left(-2m(\cos x - m\sin x)f(x) + f(x)(m\cos x + \sin x) + f'(x)(\cos x - m\sin x) \right)$

$\frac{e^{-2mx}}{(\cos x - m\sin x)^2} \left(f(x)(-m\cos x + (2m^2+1)\sin x) + f'(x)(\cos x - m\sin x) \right)$

$\cos x + m\sin x$ to cancel this $\cos x$

$\therefore \frac{e^{-2mx}}{(\cos x - m\sin x)} \left((2m^2+m+1) \sin^2 x \cdot dx \right) \leftarrow$ for $f(x) = \cos x + m\sin x$

$\therefore \int \frac{e^{-2mx} \sin^2 x \cdot dx}{(\cos x - m\sin x)^2} = \boxed{\frac{e^{-2mx}}{(m^2+1)} \frac{(\cos x + m\sin x)}{(\cos x - m\sin x)}} + C$

Ans 7 $\int e^{x\sin x+\cos x}(x^2\cos x+1)+\int e^{x\sin x+\cos x}\left(\frac{x^2\cos^2 x-x\sin x+\cos x}{x^2\cos^2 x}\right)$

$= e^{x\sin x+\cos x}\cdot x + -\int e^{x\sin x+\cos x}\left(\frac{x\cos x}{x\cos x}+\frac{x\sin x-\cos x}{x^2\cos^2 x}\right)$

as $\frac{d}{dx}(x\sin x+\cos x)=x\cos x$

$= e^{x\sin x+\cos x}\cdot x - e^{x\sin x+\cos x}\cdot\left(\frac{1}{x\cos x}\right)$

$= \boxed{e^{x\sin x+\cos x}\left(x-\frac{1}{x\cos x}\right)+c}$

Ans 8 $\int\frac{\operatorname{cosec}^2 x\, dx}{(\operatorname{cosec} x+\cot x)^{\frac{9}{2}}} = \int\frac{\sin x^{\frac{5}{2}}}{(1+\cos x)^{\frac{9}{2}}} = \int\frac{2^{\frac{5}{2}}\sin\frac{x}{2}^{\frac{5}{2}}\cos\frac{x}{2}^{\frac{5}{2}}}{2^{\frac{9}{2}}\cos^9\frac{x}{2}}$

$=\int\frac{1}{4}\left(\tan\frac{x}{2}\right)^{\frac{5}{2}}\sec^4\frac{x}{2}\cdot dx$

$\tan\frac{x}{2}=t \qquad \frac{1}{2}\sec^2\frac{x}{2}\cdot dx=dt$

$=\int\frac{(1+t^2)}{2\, t^{\frac{5}{2}}}\frac{\sec^2\frac{x}{2}\cdot dx}{2} = \int\frac{1}{2}\frac{(1+t^2)}{t^{\frac{5}{2}}}\cdot dt$

$=\frac{1}{2}\int\left(t^{-\frac{5}{2}}+t^{-\frac{1}{2}}\right)\cdot dt = \frac{1}{2}\left(\frac{-3}{2}t^{-\frac{3}{2}}+2t^{\frac{1}{2}}\right)+c$

$=\frac{1}{4}\left(\frac{t^2-3}{t^{\frac{3}{2}}}\right)+c \qquad t=\tan\frac{x}{2}$

$=\frac{1}{4}\left(\frac{\tan^2\frac{x}{2}-3}{\left(\tan\frac{x}{2}\right)^{\frac{3}{2}}}\right)+c$

$=\int\frac{1}{2}t^{\frac{5}{2}}(1+t^2)\cdot dt$

$=\frac{1}{2}\int\left(t^{\frac{5}{2}}+t^{\frac{9}{2}}\right)\cdot dt$

$=\frac{1}{2}\left(\frac{2}{7}\tan^{\frac{7}{2}}\frac{x}{2}+\frac{2}{11}\tan^{\frac{11}{2}}\frac{x}{2}\right)$

$=\int\frac{1}{7}\left(\tan\frac{x}{2}\right)^{\frac{7}{2}}+\frac{2}{11}\left(\tan\frac{x}{2}\right)^{\frac{11}{2}}+c$

Ans 6 (II) $\int \frac{\sin^2 x \, dx}{\left(e^{mx}(\cos x - m\sin x)\right)^2}$ $\frac{d}{dx}\left(e^{mx}(\cos x - m\sin x)\right) = e^{mx}(-m^2-1)\sin x$

$= \frac{-1}{m^2+1} \int \frac{\sin x}{e^{mx}} \frac{\left(-e^{mx} \cdot \sin x(m^2+1)\right) \cdot dx}{\left(e^{mx}(\cos x - m\sin x)\right)^2}$

By parts

$= \frac{1}{m^2+1} \left(\frac{e^{mx}\cos x - me^{mx}\sin x}{e^{2mx}} \right.$

$= \frac{-1}{m^2+1} \left(\frac{\sin x}{e^{mx}} \left(\frac{1}{e^{mx}(\cos x - m\sin x)} \right) \right)$

$\frac{e^{-2mx}(\cos x + m\sin)}{2m^2+m+1}$

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