

Q. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$ equals to.

$$\text{Let } \sec x + \tan x = t \text{ --- (1)} \Rightarrow (\sec x \tan x + \sec^2 x) dx = dt$$

$$\Rightarrow \sec x - \tan x = \frac{1}{t} \text{ --- (3)} \quad \sec x (\sec x + \tan x) dx = dt \text{ --- (2)}$$

$$\therefore \sec x dx = \frac{dt}{t} \quad (\text{from (1) \& (2)})$$

$$\frac{1}{2} \left(t + \frac{1}{t} \right) = \sec x \quad (\text{from (1) \& (3)})$$

$$\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx = \frac{1}{2} \int \frac{\left(t + \frac{1}{t} \right) dt}{t^{9/2} \cdot t} = \frac{1}{2} \int \left(t^{-9/2} + t^{-13/2} \right) dt$$

$$= \frac{1}{2} \left[\frac{t^{-7/2}}{-7/2} + \frac{t^{-11/2}}{-11/2} \right] = -\frac{t^{-7/2}}{7} - \frac{t^{-11/2}}{11} + C$$

$$= -\frac{1}{t^{11/2}} \left[\frac{1}{11} + \frac{t^2}{7} \right] + C$$

$$= \boxed{-\frac{1}{(\sec x + \tan x)^{11/2}} \left[\frac{1}{11} + \frac{1}{7} (\sec x + \tan x)^2 \right] + C}$$