Methods of integration

1. Substitution
2. Integration by parts
3. Trigonometric
4. Algebra
5. Partial fraction
6. Reduction formula.

1. Integration by substitution

Case-I :- $\int f(x).f'(x).dx$, $\int \frac{f'(x)}{f(x)}.dx$, $\int (f(x))^n f'(x).dx$, $\int \frac{f'(x).dx}{(f(x))^n}$

→ We put $f(x) = t$, diff both side with respect to x

$f'(x).dx = dt \leftarrow f'(x) = \frac{dt}{dx}$

$\Rightarrow \int t.dt, \int \frac{dt}{t}, \int t^n.dt, \int \frac{dt}{t^n}$

$\Rightarrow \frac{t^2}{2} + C \qquad \ln t + C$

$= \frac{1}{2}(f(x))^2 + C, \ \ln f(x) + C$

Eg. $\int \cot x \cdot dx = \int \frac{\cos x}{\sin x} \cdot dx$ put $\sin x = t$ $= \int \frac{dt}{t} = \ln \sin x + c$

$\int \sec x \cdot dx = \int \frac{\sec x \cdot (\sec x + \tan x)}{(\sec x + \tan x)} = \int \frac{dt}{t} = \ln(\sec x + \tan x) + c$

$\boxed{t = (\sec x + \tan x)}$

Case II :- $\int g(f(x)) \cdot f'(x) \cdot dx$, $\int \frac{f'(x) \cdot dx}{g(f(x))}$, $\int \frac{f'(x) \cdot dx}{g(f'(x))}$

Eg $\int \cos(e^x) \cdot e^x \cdot dx$ $e^x = t$

$\int \cos t \cdot dt = \boxed{\sin e^x + c}$

Case III :- $\int \frac{f'(x)}{a + (f(x))^n}$ a is constant

Eg $\int \frac{\sqrt{x} \cdot dx}{1 + x^3} = \int \frac{\sqrt{x}}{1 + (x^{\frac{3}{2}})^2} dx$: put $x^{\frac{3}{2}} = t$

$= \int \frac{2 \cdot dt}{3(1 + t^2)} = \frac{2}{3} \tan^{-1} t = \boxed{\frac{2}{3} \tan^{-1} x^{\frac{3}{2}} + c}$

Case IV :- $\int \sqrt{a^2 - x^2} \cdot dx$, $\int \frac{dx}{\sqrt{a^2 - x^2}}$, $\int \frac{dx}{a^2 - x^2}$

Put $x = a \sin\theta$ or $x = a \cos\theta$

$x = a \sin\theta \Rightarrow dx = a \cos\theta \cdot d\theta$

$\int \frac{a \cos\theta \cdot d\theta}{\sqrt{a^2 - a^2 \sin^2\theta}} = \int \frac{a \cos\theta \cdot d\theta}{a \cos\theta} = \theta + c = \boxed{\sin^{-1}\left(\frac{x}{a}\right) + c}$

$$\int \frac{dx}{a^2-x^2} = -\int \frac{dx}{(x-a)(x+a)} = \frac{-1}{2a}\int \frac{(x+a)-(x-a)}{(x+a)(x-a)}\cdot dx$$

→ We are only talking about one of the methods and their may be other method possible.

If ⇒ $\int \sqrt{a^2-(cx+d)^2}\cdot dx$, $\int \frac{dx}{\sqrt{a^2-(cx+d)^2}}$, $\int \frac{dx}{a^2-(cx+d)^2}$

Put : $cx+d = a\sin\theta$ or $cx+d = a\cos\theta$

Case V. $\int \sqrt{a^2+x^2}\cdot dx$, $\int \frac{dx}{\sqrt{a^2+x^2}}$, $\int \frac{dx}{a^2+x^2}$

Put $x = a\tan\theta$ or $x = a\cot\theta$

If → $\int \sqrt{a^2+(cx+d)^2}\cdot dx$, $\int \frac{dx}{\sqrt{a^2+(cx+d)^2}}$, $\int \frac{dx}{a^2+(cx+d)^2}$

Put $cx+d = a\tan\theta$ or $cx+d = a\cot\theta$

Case VI $\int \sqrt{x^2-a^2}\cdot dx$, $\int \frac{dx}{\sqrt{x^2-a^2}}$, $\int \frac{dx}{x^2-a^2}$

Put $x = a\sec\theta$ or $x = a\operatorname{cosec}\theta$

Case VII : $\int \sqrt{\frac{a-x}{a+x}}\cdot dx$ (i) Put $x = a\cos 2\theta$

(ii) $\int \frac{\sqrt{a-x}}{\sqrt{a+x}} \times \sqrt{\frac{a-x}{a-x}}\cdot dx = \int \frac{a-x}{\sqrt{a^2-x^2}}\cdot dx$

$$= \int \frac{a\,dx}{\sqrt{a^2-x^2}} - \int \frac{x\cdot dx}{\sqrt{a^2-x^2}}$$

(iii) Put $\frac{a-x}{a+x} = t^2$

* If numerator & denominator are algebraic functions then take out max power of x, then it may change into $f(x)$ & $f'(x)$ most of the times

* If $\int\sqrt{f(x)}\,.dx$, then one method can be put $f(x) = t^2$ then solve

$$\frac{a-x}{a+x} = \frac{t^2}{1} \Rightarrow \frac{2a}{-2x} = \frac{t^2+1}{t^2-1} \quad \frac{x}{a} = -\frac{(t^2-1)}{(t^2+1)}$$

$$\frac{dx}{a} = -\frac{((t^2+1)\,2t - (t^2-1)\,2t)\,.dt}{(t^2+1)^2}$$

$$dx = \frac{-4at\,.dx}{(t^2+1)^2}$$ Its integration from case VIII

$$Ex = \int t\,\frac{-4at}{(t^2+1)^2}\,.dt = -4a\int\frac{t^2}{(t^2+1)^2}\,.dt$$

Case VIII Algebraic twins here k is constant

$$\int\frac{x^2}{x^4+kx^2+1}\,.dx,\ \int\frac{dx}{x^4+kx^2+1},\ \int\frac{x^2\,dx}{x^4+1},\ \int\frac{dx}{x^4+1}$$

$$\int\frac{x^2}{x^4+kx^2+1}\,.dx = \frac{1}{2}\int\frac{2x^2}{x^4+kx^2+1}\,.dx = \frac{1}{2}\int\frac{(x^2+1)+(x^2-1)}{x^4+kx^2+1}\,.dx$$

$$= \frac{1}{2}\int\frac{(x^2+1)}{x^4+kx^2+1}\,.dx + \frac{1}{2}\int\frac{x^2-1}{x^4+kx^2+1}\,.dx$$

$$= \frac{1}{2}\int\frac{1+\frac{1}{x^2}}{(x-\frac{1}{x})^2+k+2} + \frac{1}{2}\int\frac{(1-\frac{1}{x^2})\,.dx}{(x+\frac{1}{x})^2+k-2}$$

$$Eg:- \int\frac{5x^4+4x^5}{(x^5+x+1)^2} = \int\frac{x^5\,.(\frac{5}{x}+4)}{x^{10}\,(1+\frac{1}{x^4}+\frac{1}{x^5})^2} = \int\frac{\frac{1}{x^5}(\frac{5}{x}+4)\,.dx}{(1+\frac{1}{x^4}+\frac{1}{x^5})^2}$$

$$\int \frac{\frac{4}{x^5}+\frac{5}{x^6}}{\left(1+\frac{1}{x^4}+\frac{1}{x^5}\right)^2}\,dx \qquad \frac{4}{x^5}+\frac{5}{x^6}\,dx = dt$$

$$= -\int \frac{dt}{t^2} = \frac{1}{t}+C = \frac{1}{1+\frac{1}{x^4}+\frac{1}{x^5}}+C$$

Questions

Q.1 $\int \frac{x^3 \tan^{-1} x^4 \cdot dx}{1+x^8}$ $\quad x^4 = t$, $\quad 4x^3 \cdot dx = dt$

$= \frac{dt \tan^{-1} t}{4 \; (1+t^2)}$ $\quad \therefore \tan^{-1} t = y$, $\quad \frac{dt}{1+t^2} = dy$

$= \int \frac{y\,dy}{4} = \frac{y^2}{8} + C = \boxed{\frac{(\tan^{-1} x^4)^2}{8} + c}$

Q.2 $\int \frac{x \cdot dx}{1+x^4}$ $\quad x^2 = t$, $\quad 2x \cdot dx = dt$

$= \int \frac{dt}{2(1+t^2)} = \frac{\tan^{-1} t}{2} + C = \frac{\tan^{-1} x^2}{2} + C$

Q.3 $\int \frac{dx}{x(x^3+1)} = \int \frac{dx}{x^4\left(1+\frac{1}{x^3}\right)}$ $\quad 1+\frac{1}{x^3} = t$, $\quad -\frac{3}{x^4} \cdot dx = dt$

$\int \frac{-dt}{3t} = \frac{-\ln t}{3} + C$

$= \boxed{\frac{-\ln\left(1+\frac{1}{x^3}\right)}{3} + C}$

Q.4 $\int \frac{(2x+3)\,dx}{x(x+1)(x+2)(x+3)} = \frac{(2x+3)\,dx}{(x^2+3x)(x^2+3x+2)}$ $\therefore x^2+3x+2=t$

$(2x+3)\,dx = dt$

$= \frac{dt}{(t-2)(t)} = \int \frac{1}{2}\left(\frac{1}{t-2} - \frac{1}{t}\right).dt$

$= \frac{1}{2}(\ln t-2 - \ln t) = \frac{1}{2} \ln \frac{t-2}{t} = \boxed{\frac{1}{2} \ln \frac{x^2+3x}{x^2+3x+2} + c}$

Q.5 $\int \frac{\ln \ln x}{x \ln x}.dx$

$\ln \ln x = t$

$\frac{dx}{\ln x . x} = dt$

$= \int t.dt = \frac{t^2}{2} + c = \boxed{\frac{(\ln \ln x)^2}{2} + c}$

Q.6 $\frac{(x+1)(x+\ln x)^2}{x}.dx$

Let $x + \ln x = t$

$1 + \frac{1}{x} = dt$

$\therefore \int t^2.dt = \frac{t^3}{3} + c$

$= \boxed{\frac{(x+\ln x)^3}{3} + c}$

Q.7 $\int \frac{(\cos x - \sin x).dx}{e^x + \sin x} = \frac{e^{-x}(\cos x - \sin x)}{(1+e^{-x}\sin x)}$

$\therefore 1+e^{-x}\sin x = t$ $\therefore -e^{-x}\cos x - e^{x}\sin x = dt$

$= \int \frac{dt}{t} = \ln t + c = \boxed{\ln(1+e^{-x}\sin x) + c}$

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Q-8 $\int \frac{(\cos x + x\sin x)}{x(x+\cos x)} \cdot dx = \int \frac{(\cos x + x\sin x) \cdot dx}{x^2\left(1+\frac{\cos x}{x}\right)}$.

M-I $\therefore 1 + \frac{\cos x}{x} = t \quad \frac{-\sin x \cdot x - \cos x}{x^2} = dt$

$= \int \frac{-dt}{t} = -\ln t + c = \boxed{-\ln\left(1+\frac{\cos x}{x}\right) + c}$

M-II $\frac{\cos x + x\sin x + x - x}{x(x+\cos x)} = \int \frac{1}{x} + \frac{\sin x - 1}{x + \cos x}$

$= \ln x + -\int \frac{dt}{t}$

$= \boxed{\ln x - \ln(x+\cos x) + C}$

Q.9 $\int \frac{(x^4+1)}{x^6+1} \cdot dx$

I $= \frac{x^4 - x^2 + 1 + x^2}{(x^2+1)(x^4-x^2+1)} = \frac{1}{x^2+1} + \frac{x^2}{x^6+1}$

$= \tan^{-1}x + \int \frac{dt}{3t} = \boxed{\tan^{-1}x + \frac{1}{3}\tan^{-1}x^3 + c}$

II $\to \int (1 - x^6 + x^{12} - x^{18} + \cdots) + (x^4 - x^{10} + x^{16} - x^{22} \cdots) \cdot dx$

$\Rightarrow \left(x - \frac{x^7}{7} + \frac{x^{13}}{13} - \frac{x^{19}}{19} \cdots\right) + \left(\frac{x^5}{5} - \frac{x^{11}}{11} + \frac{x^{17}}{17} \cdots\right)$

$\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} \cdots$

$\frac{\tan^{-1}x^3}{3} = \frac{1}{3}\left((x^3) - \frac{(x^3)^3}{3} + \frac{(x^3)^5}{5} \cdots\right)$

$= \tan^{-1}x + \frac{\tan^{-1}x^3}{3}$

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Q.10 $\int \frac{(\sin x + \cos x)}{9 + 16\sin 2x} \cdot dx$

$= \frac{(\sin + \cos x) \cdot dx}{25 - 16(\sin x - \cos x)^2}$ $\quad \therefore \sin x - \cos x = t$

$\therefore -(\cos x + \sin x) \cdot dx = dt$

$= \int \frac{dt}{25 - 16t^2} = \int \left(\frac{1}{(5-4t)} + \frac{1}{5+4t} \right) \frac{dt}{10} = \frac{\ln(5-4t) + \ln(5+4t)}{10}$

$= \frac{\ln(25 - 16t^2)}{10} + C = \frac{\ln(25 - 16(\sin x - \cos x)^2)}{10} + C$

$= \frac{\ln(9 + 16\sin 2x)}{10}$

Q.11 $\int \frac{dx}{\sqrt{x}(x+a)}$

$x^{\frac{1}{2}} = t$

$\frac{1}{2x^{\frac{1}{2}}} \cdot dx = dt$

$= \int \frac{2dt}{t^2 + a} = \frac{2}{\sqrt{a}} \tan^{-1} \frac{t}{\sqrt{a}} = \boxed{\frac{2}{\sqrt{a}} \tan^{-1} \frac{x^{\frac{1}{2}}}{a^{\frac{1}{2}}} + C}$

As $a = 9$

$\therefore \boxed{\frac{2}{3} \tan^{-1} \frac{x^{\frac{1}{2}}}{3} + c}$

Q.1 $\int \frac{dx}{(2\sin x+3\cos x)^2}$

$\frac{2}{\sqrt{13}}\sin x + \frac{3}{\sqrt{13}}\cos x =$

$= \sin\left(x+\sin^{-1}\frac{3}{\sqrt{13}}\right)$

$= \frac{1}{13\left(\sin^2\left(x+\sin^{-1}\frac{3}{\sqrt{13}}\right)\right)}$

$= \frac{\operatorname{cosec}^2\left(x+\sin^{-1}\frac{3}{\sqrt{13}}\right)}{13} = \boxed{\frac{-\cot\left(x+\sin^{-1}\frac{3}{\sqrt{13}}\right)}{13}} + c$

$= -\frac{1}{2(3+2\tan x)} + c$

Q.2 $\int \frac{(x\cos x+1)\,dx}{\sqrt{2x^3e^{\sin x}+x^2}} = \int \frac{x\cos x+1}{x\sqrt{2xe^{\sin x}+1}}$

$\therefore\ 2xe^{\sin x}+1 = t^2$

$\cancel{2}(e^{\sin x}+x\cos x e^{\sin x}) = \cancel{2}t\,dt$

$= \frac{t\,dt}{e^{\sin x}\cdot x\, t} = \int \frac{2\,dt}{t^2-1} = \int \frac{dt}{t-1} - \frac{dt}{t+1} = \ln\frac{t-1}{t+1}$

$= \boxed{\ln\frac{2xe^{\sin x}}{2xe^{\sin x}+2}} + c$

Q.3 $\int \frac{dx}{\sin x+\sec x} = \frac{2\cos x}{2+\sin 2x} = \int \frac{(\cos x-\sin x)\,dx}{2+\sin 2x} + \int \frac{(\cos x+\sin x)\,dx}{2+\sin 2x}$

$= \int \frac{(\cos x-\sin x)\,dx}{1+(\cos x+\sin x)^2} + \int \frac{\cos x+\sin x}{3-(\sin x-\cos x)^2} = \int\frac{dt}{1+t^2} + \int\frac{dp}{3-p^2}$

$= \tan^{-1}(\cos x+\sin x) + \frac{1}{2\sqrt{3}}\ln\left(\frac{\sqrt{3}+x}{\sqrt{3}-x}\right) + c$

Q.4 $\int \frac{x^4-1}{x^2(x^4+x^2+1)^{\frac{1}{2}}} = \int \frac{(x-\frac{1}{x^3})\,.dx}{(x^2+1+\frac{1}{x^2})^{\frac{1}{2}}} = \int \frac{dt}{2\sqrt{t}}$

$= \frac{2\sqrt{t}}{2} + c = \boxed{\sqrt{x^2+1+\frac{1}{x^2}} + c}$

Q.5 $\int (\sqrt{\tan x} + \sqrt{\cot x})\,.dx$

$= \cancel{\frac{\tan x + 1}{\sqrt{\cot x}}} \quad \cancel{\frac{(\sin x + \cos x)}{\sqrt{\sin x \cos x}}} = \cancel{\frac{\sin(x+\frac{\pi}{4})\,2}{\sqrt{\sin 2x}}}$

$= \cancel{\frac{\sin(x+\frac{\pi}{4}) - \cos(x+\frac{\pi}{4})}{1}} + \cancel{\sin(x+\frac{\pi}{4}) + \cos(x+\frac{\pi}{4})}$

$= \frac{\sqrt{2}(\sin x + \cos x)}{\sqrt{2\sin x\cos x}} = \frac{\sin x + \cos x}{\sqrt{1-(\sin x - \cos x)^2}} \quad \sin x - \cos x = t$

$= \int \frac{\sqrt{2}\,dt}{\sqrt{1-t^2}} = \sqrt{2}\sin^{-1}(t) + c = \sqrt{2}\sin^{-1}(\sin x - \cos x) + c$

Q.6 $\int \frac{x^{2009}}{(1+x^2)^{1006}}\,.dx = \frac{dx}{x^3(1+\frac{1}{x^2})^{1006}} = \int -\frac{dt}{2(t^{1006})}$

$= \frac{+1}{2\times 1005}\, t^{-1005} = \boxed{\frac{1}{2010}\left(1+\frac{1}{x^2}\right)^{-1005} + c}$

Q.7 $\int \frac{\sin x}{\sin x + \cos x}$

$$= \frac{1}{2}\left(\frac{(\sin x+\cos x)}{\sin x+\cos x}.dx + \frac{(\sin x-\cos x)}{\sin x+\cos x}.dx\right)$$

$$= \boxed{\frac{1}{2}\left(x - \ln(\sin x+\cos x)\right)+C}$$

Q.8 $\int \sqrt{\dfrac{\cos x-\cos^3 x}{1-\cos^3 x}}$

Q.9 $\int \dfrac{dx}{(x^2+2x+10)^3}$

Q.10 $\int \left(\dfrac{1+x^2\ln x}{x+x^2\ln x}\right).dx$

Ans 8 $\sin x\sqrt{\dfrac{\cos x}{1-\cos^3 x}}$ $\qquad 1-\cos^3 x = t$

$\qquad +3\sin x\cos^2 x = dt$

$= \dfrac{dt}{3}\sqrt{\dfrac{1}{t(1-t)}}$ $\qquad \therefore 1-t = p^2$

$\qquad -dt = 2p.dp$

$= \dfrac{-2p.dp}{3p\sqrt{1-p^2}} = -\dfrac{2}{3}\sin^{-1}p = \boxed{-\dfrac{2}{3}\sin^{-1}\cos^{\frac{3}{2}}x + C}$

Ans 9 $= \int \dfrac{dx}{((x+1)^2+3^2)^3}$ $\qquad \therefore x+1 = 3\tan\theta$

$\qquad dx = 3\sec^2\theta.d\theta$

$= \dfrac{3\sec^2\theta.d\theta}{3^6\sec^6\theta} = \dfrac{1}{3^5}(\cos^2\theta - \cos^2\theta\sin^2\theta).d\theta = \dfrac{1}{3^5}\left(\dfrac{1+\cos 2x}{2} + \dfrac{\cos 4x-1}{8}\right)$

$\boxed{\dfrac{1}{944}\left(2\sin 2\theta + \sin 4\theta + \dfrac{3\theta}{8} + C\right)}$

where $\theta = \tan^{-1}\dfrac{(x+1)}{3}$

$= \dfrac{\int(\frac{3}{8}\cos 2\theta + \cos 4\theta)d\theta}{2.3^5} = \dfrac{\frac{\sin 2\theta}{2} + \frac{\sin 4\theta}{4} + 3\theta}{2.3^5} = \boxed{\dfrac{1}{2^2.3^5}\left(\sin 2\tan^{-1}\dfrac{(x+1)}{3} + \dfrac{\sin 4\tan^{-1}\frac{(x+1)}{3}}{2} + \dfrac{3\theta}{2}\right)}$

Ans 10 $\int \left(\dfrac{1-x}{x^2(\frac{1}{x}+\ln x)} + 1\right).dx = \int \dfrac{-dt}{t} + x = \boxed{-\ln\left(\dfrac{1}{x}+\ln x\right) + x + C}$

Case-I – $\int \sqrt{\tan x}\,.dx$, $\int \sqrt{\cot x}\,.dx$

Put $\tan x = t^2$, put $\cot x = t^2$

Case II – $\int (x \pm \sqrt{x^2 \pm a^2})^n\,.dx$

Put $x \pm \sqrt{x^2 \pm a^2} = t$

Eg– $\int (x + \sqrt{x^2+a^2})^n\,.dx$

$x + \sqrt{x^2+a^2} = t$

$x^2 + a^2 = x^2 + t^2 - 2xt$

$x = \frac{t^2 - a^2}{2t}$

$dx = \frac{1}{2}\left(1 + \frac{a^2}{t^2}\right).dt$

$\boxed{= \frac{1}{2}\int t^n \left(1 + \frac{a^2}{t^2}\right).dt}$

Case III $\int \sqrt{(x-a)(b-x)}\,.dx$, $\int \sqrt{\frac{(x-a)}{(b-x)}}\,.dx$

for both

(I) Put $x = a\cos^2\theta + b\sin^2\theta$

(II) $\int \sqrt{(x-2)(4-x)} = \int \sqrt{4x - x^2 - 8 + 2x} = \int \sqrt{-(x^2 - 6x + 8)}$

$= \int \sqrt{1 - (x-3)^2}\,.dx$

(I) Put $\frac{x-a}{b-x} = t^2$

(II) $\int \frac{(x-a).dx}{\sqrt{(x-a)(b-x)}}$

Case-IV $\int \sqrt{(x-a)(x-b)}\,dx$, $\int \sqrt{\frac{x-a}{x-b}}\,dx$

(I) Put $x = a\sec^2\theta - b\tan^2\theta$

for both

(II) Making perfect square

Q.1 $\int (x^{3m}+x^{2m}+x^{m})(2x^{2m}+3x^{m}+6)^{\frac{1}{m}}\,dx$.

$\int (x^{3m-1}+x^{2m-1}+x^{m-1})(x^{m})^{\frac{1}{m}}(2x^{2m}+3x^{m}+6)^{\frac{1}{m}}\,dx$

$\int (x^{3m-1}+x^{2m-1}+x^{m-1})(\underbrace{2x^{3m}+3x^{2m}+6x^{m}}_{t})^{\frac{1}{m}}\,dx$

$6m(x^{3m-1}+x^{2m-1}+x^{m-1})\,dx = dt$

$\int \frac{dt}{6m}t^{\frac{1}{m}} = \frac{1}{6(m+1)}t^{\frac{m+1}{m}} = \boxed{\frac{(2x^{3m}+3x^{2m}+6x^{m})^{\frac{m+1}{m}}}{6(m+1)}+c}$

Q.2 $\frac{dx}{\sqrt{\sin^3 x \sin(x+\alpha)}}$

Q.3 $\frac{dx}{\sin^3 x+\cos^3 x}$

Q.4 $\int \frac{dx}{\sqrt{1-\tan^2 x}}$

Ans 3 M-II

$\frac{2\,dx}{(\sin x+\cos x)(2-\sin 2x)} = \frac{\sqrt{2}\,dx}{\sin(x+\frac{\pi}{4})(2-\sin 2x)}$

$x+\frac{\pi}{4} = t \quad dx = dt$

$= \sqrt{2}\int \frac{dt}{\sin t(2-\sin(2t-\frac{\pi}{2}))} = \sqrt{2}\int \frac{dt}{\sin t(2+\cos 2t)}$

$= \sqrt{2}\int \frac{\sin t}{(1-\cos^2 t)}$

Ans 2 $\int \frac{dx}{\sqrt{\sin^3 x \sin(x+2)}} = \int \frac{dx}{\sin^2 x \sqrt{\frac{\sin(x+2)}{\sin x}}} = \int \frac{dx \operatorname{cosec}^2 x}{\sqrt{\cos 2 + \cot x \sin 2}}$

$\cos 2 + \cot x \sin 2 = t$

$-\operatorname{cosec}^2 x \sin 2 \, dx = dt$

$= \int \frac{-dt}{\sin 2 \sqrt{t}} = \frac{-2\sqrt{t}}{\sin 2} = \boxed{-2(\operatorname{cosec} 2)\sqrt{\frac{\sin(x+2)}{\sin x}} + C}$

Ans 3 $\int \frac{dx}{\sin^3 x + \cos^3 x} = \int \frac{dx}{(\sin x + \cos x)(1 - \sin x \cos x)} = \int \frac{2}{(\sin x + \cos x)\left(\frac{1}{2} - \frac{(\sin x - \cos x)^2}{2}\right)}$

$\int \frac{2 \, dx}{(\sin x + \cos x)(1 + (\sin x - \cos x)^2)}$ $\therefore$ $\sin x - \cos x = t$

$(\cos x + \sin x) \, dx = dt$

$dx = \frac{dt}{\sin x + \cos x}$

$= \int \frac{2\,dt}{(\sin x + \cos x)^2 (1 + t^2)} = \frac{2\,dt}{(2 - (\sin x - \cos x)^2)(1 + t^2)}$

$= \int \frac{2\,dt}{(2 - t^2)(1 + t^2)} = \left(\frac{dt}{(1+t^2)} + \frac{dt}{2 - t^2}\right)\frac{2}{3}$

$= \left(\tan^{-1} t + \frac{1}{2\sqrt{2}} \ln \frac{\sqrt{2}+t}{\sqrt{2}-t}\right)\frac{2}{3} + C$ where $\boxed{t = \sin x - \cos x}$

Ans 4 $\int \frac{dx}{\sqrt{1 - \tan^2 x}} = \int \frac{dx \cos x}{\sqrt{\cos^2 x - \sin^2 x}} = \int \frac{dx \cos x}{\sqrt{1 - 2\sin^2 x}}$ $\therefore$ put $\sin x = t$, $\cos x \, dx = dt$

$= \int \frac{dt}{\sqrt{1 - 2t^2}} = \frac{1}{\sqrt{2}} \sin^{-1} \sqrt{2} t + C$

$= \boxed{\frac{1}{\sqrt{2}} \sin^{-1} \sqrt{2} \sin x + C}$

Q. 5 $\int \frac{dx}{\tan x+\cot x+\sec x+\operatorname{cosec} x}$

$= \int \frac{\sin x\cos x}{1+\sin x\cos x} = \int \frac{2\sin\frac{x}{2}\cos\frac{x}{2}\cos x\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{2\cos\frac{x}{2}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}dx$

$= \int \sin\frac{x}{2}\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right).dx$

$= \int\left(\frac{\sin x}{2}+\frac{\cos x-1}{2}\right).dx = \boxed{-\frac{\cos x}{2}+\frac{\sin x}{2}-\frac{x}{2}+c}$

Q. 6 $\int \frac{x^2+1}{x\sqrt{x^2+2x-1}\sqrt{1-x^2-x}}$

$= \int \frac{1+\frac{1}{x^2}}{\sqrt{\left(x-\frac{1}{x}\right)+2}\sqrt{\frac{1}{x}-x-1}}$ $\quad \therefore x-\frac{1}{x}+2=t^2$

$\left(1+\frac{1}{x^2}\right)dx = 2t\,dt$

$= \int \frac{2.dt}{\sqrt{1-t^2}} = 2\sin^{-1}t+c$

$= \boxed{2\sin^{-1}\left(\sqrt{x-\frac{1}{x}+2}\right)+c}$