

## Questions

$$Q.1 \int \frac{x^3 \tan^{-1} x^4}{1+x^8} dx \quad x^4 = t$$

$$4x^3 dx = dt$$

$$= \frac{dt \tan^{-1} t}{4(1+t^2)}$$

$$\therefore \tan^{-1} t = y$$

$$\frac{dt}{1+t^2} = dy$$

$$= \int \frac{y dy}{4} = \frac{y^2}{8} + C$$

$$= \frac{(\tan^{-1} x^4)^2}{8} + C$$

$$Q.2 \int \frac{x \cdot dx}{1+x^4}$$

$$x^2 = t$$

$$2x \cdot dx = dt$$

$$= \int \frac{dt}{\sqrt{2}(1+t^2)}$$

$$= \frac{\tan^{-1} t}{2} + C = \frac{\tan^{-1} x^2}{2} + C$$

Q.3

$$\int \frac{dx}{x(x^3+1)} = \int \frac{dx}{x^4 \left(1 + \frac{1}{x^3}\right)}$$

$$\frac{1+x}{x^3} = t$$

$$-\frac{3 \cdot dx}{x^4} = dt$$

$$\int \frac{-dt}{3t} = -\frac{\ln t}{3} + C$$

$$= \frac{-\ln\left(1 + \frac{1}{x^3}\right)}{3} + C$$

Q.4  $\int \frac{(2x+3) \cdot dx}{x(x+1)(x+2)(x+3)} = \frac{(2x+3) \cdot dx}{(x^2+3x)(x^2+3x+2)} \quad \therefore x^2+3x+2 = t$   
 $(2x+3) \cdot dx = dt$

$$= \frac{dt}{(t-2)(t)} = \frac{1}{2} \int \left( \frac{1}{t-2} - \frac{1}{t} \right) \cdot dt$$

$$= \frac{1}{2} (\ln t - 2 - \ln t) = \frac{1}{2} \ln t - 2 = \boxed{\frac{1}{2} \ln \frac{x^2+3x+2}{x^2+3x} + c}$$

Q.5  $\int \frac{\ln \ln x \cdot dx}{x \ln x} \quad \ln \ln x = t$   
 $\frac{dx}{\ln x \cdot x} = dt$

$$= \int t \cdot dt = \frac{t^2}{2} + c = \boxed{\frac{(\ln \ln x)^2}{2} + c}$$

Q.6  $\int \frac{(x+1)(x+\ln x)^2}{x} \cdot dx$

Let  $x + \ln x = t$   
 $1 + \frac{1}{x} = dt$

$$\therefore \int t^2 \cdot dt = \frac{t^3}{3} + c$$

$$= \boxed{\frac{(x + \ln x)^3}{3} + c}$$

Q.7  $\int \frac{(\cos x - \sin x) \cdot dx}{e^x + \sin x} = \int \frac{e^{-x} (\cos x - \sin x)}{1 + e^x \sin x}$

$$\therefore 1 + e^{-x} \sin x = t \quad \therefore -e^{-x} \cos x - e^x \sin x = dt$$

$$= \int \frac{dt}{t} = \ln t + c = \boxed{\ln(1 + e^{-x} \sin x) + c}$$

Q-8  $\int \frac{(\cos x + x \sin x) \cdot dx}{x(x + \cos x)} = \int \frac{(\cos x + x \sin x) \cdot dx}{x^2 \left(1 + \frac{\cos x}{x}\right)}$

M-I

$\therefore 1 + \frac{\cos x}{x} = t \quad \frac{-\sin x \cdot x - \cos x}{x^2} = dt$

$= \int \frac{-dt}{t} = -\ln t + C = \boxed{-\ln\left(1 + \frac{\cos x}{x}\right) + C}$

M-II

$\frac{\cos x + x \sin x + x - x}{x(x + \cos x)} = \int \frac{1}{x} + \frac{-\sin x - 1}{x + \cos x}$

$= \ln x + \int \frac{dt}{t}$

$= \boxed{\ln x - \ln(x + \cos x) + C}$

Q.9  $\int \frac{(x^4 + 1) \cdot dx}{x^6 + 1}$

I

$= \frac{x^4 - x^2 + 1 + x^2}{(x^2 + 1)(x^4 - x^2 + 1)} = \frac{1}{x^2 + 1} + \frac{x^2}{x^6 + 1}$

$= \tan^{-1} x + \int \frac{dt}{3t} = \boxed{\tan^{-1} x + \frac{1}{3} \tan^{-1} x^3 + C}$

II

$\rightarrow \int (1 - x^6 + x^{12} - x^{18} + \dots) \cdot (x^6 - x^{10} + x^{16} - x^{22} + \dots) dx$

$= \int \left( x - \frac{x^7}{7} + \frac{x^{13}}{13} - \frac{x^{19}}{19} + \dots \right) + \left( \frac{x^5}{5} - \frac{x^{11}}{11} + \frac{x^{17}}{17} + \dots \right) dx$

$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$

$\frac{\tan^{-1} x^3}{3} = \frac{1}{3} \left( x^3 - \frac{(x^3)^2}{3} + \frac{(x^3)^5}{5} - \dots \right)$

$= \tan^{-1} x + \frac{\tan^{-1} x^3}{3}$

Q.10  $\int \frac{(\sin u + \cos u) \cdot du}{9 + 16 \sin 2u}$

$$= \frac{(\sin u + \cos u) \cdot du}{25 - 16(\sin u - \cos u)^2}$$

$$\therefore \sin u - \cos u = t$$

$$\therefore -(\cos u + \sin u) \cdot du = dt$$

$$= \int \frac{dt}{25 - 16t^2} = \int \left( \frac{1}{(5-4t)} + \frac{1}{(5+4t)} \right) \frac{dt}{10} = \frac{\ln(5-4t) + \ln(5+4t)}{10}$$

$$= \frac{\ln(25 - 16t^2)}{10} + C = \frac{\ln(25 - 16(\sin u - \cos u)^2)}{10} + C$$

$$= \frac{\ln(9 + 16 \sin 2u)}{10}$$

Q.11  $\int \frac{dx}{\sqrt{x(x+a)}}$

$$x^{\frac{1}{2}} = t$$

$$\frac{1}{2} x^{-\frac{1}{2}} \cdot dx = dt$$

$$\therefore \int \frac{2dt}{t^2 + a} = \frac{2 \tan^{-1} \frac{t}{\sqrt{a}}}{\sqrt{a}} = \frac{2 \tan^{-1} \frac{x^{\frac{1}{2}}}{\sqrt{a}}}{\sqrt{a}} + C$$

As  $a = 9$

$$\therefore \frac{2}{3} \tan^{-1} \frac{x^{\frac{1}{2}}}{3} + C$$

$$Q.1 \int \frac{dx}{(2\sin x + 3\cos x)^2}$$

$$\frac{2\sin x + 3\cos x}{\sqrt{13}} = \sin\left(x + \sin^{-1}\frac{3}{\sqrt{13}}\right)$$

$$= \frac{1}{13 \left(\sin^2\left(x + \sin^{-1}\frac{3}{\sqrt{13}}\right)\right)}$$

$$= \frac{\operatorname{cosec}^2\left(x + \sin^{-1}\frac{3}{\sqrt{13}}\right)}{13}$$

$$= \frac{-\cot\left(x + \sin^{-1}\frac{3}{\sqrt{13}}\right)}{13} + C$$

$$= \frac{-1}{2(3+2\tan x)} + C$$

$$Q.2 \int \frac{(x\cos x + 1) dx}{\sqrt{2x^3 e^{\sin x} + x^2}}$$

$$= \int \frac{x\cos x + 1}{x^2 \sqrt{2x e^{\sin x} + 1}}$$

$$\therefore 2x e^{\sin x} + 1 = t^2$$

$$2(e^{\sin x} + x \cos x e^{\sin x}) \frac{dx}{dt} dt$$

$$= \frac{t dt}{e^{\sin x} x t} = \frac{2 dt}{t^2 - 1} = \int \frac{dt}{t-1} - \int \frac{dt}{t+1} = \ln|t-1| - \ln|t+1|$$

$$= \frac{\ln 2x e^{\sin x} + 1}{2x e^{\sin x} + 2} + C$$

$$Q.3 \int \frac{dx}{\sin x + \sec x}$$

$$= \frac{2\cos x}{2 + \sin 2x}$$

$$= \int \frac{(\cos x - \sin x) dx}{2 + \sin 2x} + \int \frac{(\cos x + \sin x) dx}{2 + \sin 2x}$$

$$= \int \frac{(\cos x - \sin x) dx}{1 + (\cos x + \sin x)^2} + \int \frac{\cos x + \sin x}{3 - (\sin x - \cos x)^2}$$

$$= \int \frac{dt}{1+t^2} + \int \frac{dt}{3-t^2}$$

$$= \left[ \tan^{-1}(\cos x + \sin x) + \frac{1}{2\sqrt{3}} \ln \left| \frac{\sqrt{3} + t}{\sqrt{3} - t} \right| \right] + C$$

$$Q.4 \int \frac{x^4 - 1}{x^2(x^4 + x^2 + 1)^{\frac{1}{2}}} dx = \int \frac{\left(x - \frac{1}{x^3}\right) \cdot dx}{\left(x^2 + 1 + \frac{1}{x^2}\right)^{\frac{1}{2}}} = \int \frac{dt}{2\sqrt{t}}$$

$$= \frac{2\sqrt{t}}{2} + c = \boxed{\sqrt{x^2 + 1 + \frac{1}{x^2}} + c}$$

$$Q.5 \int (\sqrt{\tan u} + \sqrt{\cot u}) \cdot du$$

$$= \frac{\sqrt{\tan u}}{\sqrt{\cot u}} + \frac{(\sqrt{\sin u \cos u})^2}{\sqrt{\sin u \cos u}} = \frac{\sin(u + \frac{\pi}{4})}{\sqrt{\sin 2u}}$$

$$= \frac{\sin(u + \frac{\pi}{4}) - \cos(u + \frac{\pi}{4})}{\sqrt{2}} + \frac{\sin(u + \frac{\pi}{4}) + \cos(u + \frac{\pi}{4})}{\sqrt{2}}$$

$$= \frac{\sqrt{2}(\sin u + \cos u)}{2 \sin u \cos u} = \frac{\sin u + \cos u}{1 - (\sin u - \cos u)^2} = \sin u - \cos u = t$$

$$= \int \frac{t dt}{\sqrt{1-t^2}} = \frac{1}{2} \sin^{-1}(t) + c = \frac{1}{2} \sin^{-1}(\sin u - \cos u) + c$$

$$Q.6 \int \frac{x^{2009} dx}{(1+x^2)^{1006}} = \frac{dx}{x^3 \left(\frac{1+x^2}{x^2}\right)^{1006}} = \int \frac{-dt}{2(t^{1006})}$$

$$= \frac{t}{2 \times 1005} t^{-1005} = \boxed{\frac{1}{2010} \left(1 + \frac{1}{x^2}\right)^{-1005} + c}$$

+ 0.1005  
 dt → 0.1005 / 1005  
 5 +

$$Q.7 \int \frac{\sin u}{\sin u + \cos u}$$

$$= \frac{1}{2} \left( \frac{(\sin x + \cos x) \cdot dx}{\sin x + \cos x} + \frac{(\sin x - \cos x) \cdot dx}{\sin x + \cos x} \right)$$

$$= \left[ \frac{1}{2} (x + \ln(\sin x + \cos x)) + C \right]$$

Q.8  $\int \sqrt{\frac{\cos x - \cos^3 x}{1 - \cos^3 x}} dx$

Q.9  $\int \frac{dx}{(x^2 + x + 10)^3}$

Q.10  $\int \left( \frac{1 + x^2 \ln x}{x + x^2 \ln x} \right) dx$

Ans 8  $\sin x \sqrt{\frac{\cos x}{1 - \cos^3 x}}$   $1 - \cos^3 x = t$   
 $+ 3 \sin^2 x \cos^2 x = dt$   
 $\frac{dt}{3 \sqrt{t(1-t)}}$   $\therefore 1-t = p^2$   
 $-dt = 2p \cdot dp$   
 $= \frac{-2p \cdot dp}{3p\sqrt{1-p^2}} = -\frac{2}{3} \sin^{-1} p = \left[ -\frac{2}{3} \sin^{-1} \cos^{\frac{2}{3}} x + C \right]$

Ans 9  $= \int \frac{-dx}{(x+1)^2 + 3^2}^3$   $\therefore x+1 = 3 \tan \theta$   
 $dx = 3 \sec^2 \theta \cdot d\theta$

$$= \frac{3 \sec^2 \theta \cdot d\theta}{3^6 \sec^6 \theta} = \frac{1}{3^5} (\cos^2 \theta - \cos^2 \theta \sin^2 \theta) d\theta = \frac{1}{3^5} \left( \frac{1 + \cos 2\theta}{2} - \frac{\cos 4\theta - 1}{4} \right)$$

$$\frac{1}{1944} \left( \frac{2 \sin 2\theta}{2} + \frac{3\theta}{3} \right) + C$$

where  $\theta = \tan^{-1} \left( \frac{x+1}{3} \right)$

$$= \frac{1}{2 \cdot 3^5} \cos 2\theta + \frac{1}{4 \cdot 3^5} \cos 4\theta = \frac{1}{2 \cdot 3^5} \left( \frac{\sin 2 \tan^{-1} \left( \frac{x+1}{3} \right)}{3} + \frac{\sin 4 \tan^{-1} \left( \frac{x+1}{3} \right)}{2} \right) + \frac{3\theta}{3 \cdot 3^5}$$

Ans 10  $\int \left( \frac{1-x}{x^2(x+\ln x)} + 1 \right) dx = \int \frac{dt}{t} + x = \left[ -\ln \left( \frac{1+\ln x}{x} \right) + x + C \right]$

Case-IV  $\int \sqrt{(x-a)/(x-b)} \cdot dx$ ,  $\int \sqrt{\frac{x-a}{x-b}} \cdot dx$

(I) Put  $x = a \sec^2 \theta - b \tan^2 \theta$   
for both

(II) Making perfect square

Q.1  $\int (x^{3m} + x^{2m} + x^m) (2x^{2m} + 3x^m + 6)^{\frac{1}{m}} dx$

$\int (x^{3m-1} + x^{2m-1} + x^{m-1}) (x)^{\frac{1}{m}} (2x^{2m} + 3x^m + 6)^{\frac{1}{m}} dx$

$\int (x^{3m-1} + x^{2m-1} + x^{m-1}) (2x^{3m} + 3x^{2m} + 6x^m)^{\frac{1}{m}} dx$

$6(x^{3m-1} + x^{2m-1} + x^{m-1}) dx = dt$

$\int \frac{dt}{6m} t^{\frac{1}{m}} = \frac{t^{\frac{1}{m}+1}}{6(m+1)} = \frac{(2x^{3m} + 3x^{2m} + 6x^m)^{\frac{m+1}{m}}}{6(m+1)}$

Q.2  $\int \frac{dx}{\sin^2 x \cos(x+\pi)}$

Ans 3 M-II

Q.3  $\int \frac{dx}{\sin^3 x + \cos^3 x}$

$\frac{2 dx}{(\sin x + \cos x)(2 - \sin^2 x)} = \frac{\sqrt{2} dx}{\sin(x + \frac{\pi}{4})(2 - \sin^2)}$

Q.4  $\int \frac{dx}{\sqrt{1 - \tan^2 x}}$

$x + \frac{\pi}{4} = t \quad dx = dt$   
 $= \frac{1}{\sqrt{2}} \int \frac{dt}{\sin t (2 - \sin^2(2t - \frac{\pi}{2}))}$   
 $= \frac{1}{\sqrt{2}} \int \frac{dt}{\sin t (1 - \cos^2 t)}$



$$\text{Ans 2} \quad \int \frac{du}{\sin^3 x \sin(x+2)} = \int \frac{du}{\sin^2 x \frac{\sin(x+2)}{\sin x}} = \int \frac{d \cos x \sec^2 x}{\sqrt{\cos 2 + \cot x \sin 2}}$$

$$\cos 2 + \cot x \sin 2 = t$$

$$-\operatorname{cosec}^2 x \sin 2 \frac{dx}{dt} = dt$$

$$= \int \frac{-dt}{\sin 2 \sqrt{t}} = \frac{-2 \sqrt{t}}{\sin 2} = \boxed{-2 \operatorname{cosec}^2 x \frac{\sin(x+2)}{\sin x} + C}$$

$$\text{Ans 3} \quad \int \frac{du}{\sin^3 + \cos^3} = \int \frac{du}{(\sin u + \cos u)(1 - \sin u \cos u)} = \int \frac{2}{(\sin u + \cos u)(1 - (\sin u \cos u)^2)}$$

$$\int \frac{2 \cdot du}{(\sin u + \cos u)(1 + (\sin u - \cos u)^2)} \quad \therefore \sin u - \cos u = t$$

$$(\cos u + \sin u) \cdot du = dt$$

$$du = \frac{dt}{\sin u + \cos u}$$

$$= \int \frac{2 dt}{(\sin u + \cos u)^2 (1 + t^2)} = \frac{2 dt}{(2 - (\sin u - \cos u)^2)(1 + t^2)}$$

$$= \int \frac{2 dt}{(2 - t^2)(1 + t^2)} = \left( \frac{dt}{1 + t^2} + \frac{dt}{2 - t^2} \right) \frac{2}{3}$$

$$= \left( \tan^{-1} t + \frac{1}{2\sqrt{2}} \ln \frac{\sqrt{2} + t}{\sqrt{2} - t} \right) \frac{2}{3} + C \quad \text{where } t = \sin u - \cos u$$

Ans 4

$$\int \frac{du}{\sqrt{1 - \tan^2 u}} = \int \frac{du \cos u}{\sqrt{\cos^2 u - \sin^2 u}} = \int \frac{du \cos u}{\sqrt{1 - 2\sin^2 u}} \quad \therefore \text{put } \sin u = t$$

$$\cos u \cdot du = dt$$

$$= \int \frac{dt}{\sqrt{1 - 2t^2}} = \frac{1}{\sqrt{2}} \sin^{-1} \sqrt{2} t + C$$

$$= \boxed{\frac{1}{\sqrt{2}} \sin^{-1} \sqrt{2} \sin u + C}$$

$$Q.5 \int \frac{dx}{\tan x + \cot x + \sec x - \csc x}$$

$$= \int \frac{\sin x \cos x}{1 + \sin x \cos x} = \int \frac{2 \sin \frac{x}{2} \cos \frac{x}{2} \cos x}{2 \cos \frac{x}{2} (\sin \frac{x}{2} + \cos \frac{x}{2}) (\cos \frac{x}{2} - \sin \frac{x}{2})}$$

$$= \int \frac{\cancel{\sin x} \cos x}{\cancel{\cos x}} = \int \frac{\sin x}{2} \left( \frac{\cos x}{2} - \frac{\sin x}{2} \right) dx$$

$$= \int \left( \frac{\sin x}{2} + \frac{\cos x - 1}{2} \right) dx = \left[ -\frac{\cos x}{2} + \frac{\sin x}{2} - \frac{x}{2} \right] + C$$

$$Q.6 \int \frac{x^2 + 1}{x \sqrt{x^2 + x - 1} \sqrt{1 - x^2 - x}}$$

$$= \int \frac{1 + \frac{1}{x}}{\sqrt{\left(\frac{x-1}{x}\right)^2} \sqrt{\frac{1-x-1}{x}}}$$

$$\therefore \frac{x-1}{x} + 2 = t^2$$

$$\left(1 + \frac{1}{x}\right) dx = 2t dt$$

$$= \int \frac{2 \cdot dt}{\sqrt{1-t^2}} = 2 \sin^{-1} t + C$$

$$= \boxed{2 \sin^{-1} \left( \sqrt{\frac{x-1}{x} + 2} \right) + C}$$