Q. $\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}\,dx$ is equal to [JEE ADVANCE 2006]

(A) $\frac{\sqrt{2x^4-2x^2+1}}{x^2}+c$ (B) $\frac{\sqrt{2x^4-2x^2+1}}{x^3}+c$

(C) $\frac{\sqrt{2x^4-2x^2+1}}{x}+c$ (D) $\frac{\sqrt{2x^4-2x^2+1}}{2x^2}+c$

Ans $\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}\,dx = \int \frac{x^2-1}{x^5\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}}\,dx$ [take common x^4 from square root term]

Now substitute $2-\frac{2}{x^2}+\frac{1}{x^4}=t$ $\therefore dt = \left(\frac{4}{x^3}-\frac{4}{x^5}\right)dx = 4\left(\frac{1}{x^3}-\frac{1}{x^5}\right)dx$

$$\int \frac{x^2-1}{x^5\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}}\,dx = \int \frac{4\left(\frac{1}{x^3}-\frac{1}{x^5}\right)dx}{4\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}} = \int \frac{dt}{4\sqrt{t}} = \frac{\sqrt{t}}{2}+c$$

$$= \frac{\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}}{2}+c = \boxed{\frac{\sqrt{2x^4-2x^2+1}}{2x^2}+c}$$ Option (D)