PART-B : AREA

4.1 Curve tracing

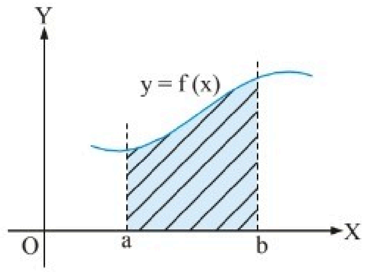
In order to find the area bounded by several curves, sometimes it is necessary to have an idea of the rough sketches of these curves. To find the approximate shape of a curve represented by the cartesian equation, the following steps are very useful.

1. Symmetry
   (a) If curve remains unaltered on replacing x by $-x$, then it is symmetrical about y-axis.
   (b) If curve remains unaltered on replacing y by $-y$, then it is symmetrical about x-axis.
2. Intersection with axes
   (a) To find points of intersection of the curve with x-axis, replace $y = 0$ in the equation of the curve and get corresponding values of x.
   (b) To find points of intersection of the curve with y-axis, replace $x = 0$ in the equation of the curve and get corresponding values of y.
3. The regions where curves does not exist
   (a) Find those values of x for which corresponding values of y do not exist.
   (b) Find intervals where $f(x)$ is positive.
4. Asymptotes
   (a) Observe where y approaches as x approaches $\pm\infty$.
   (b) If necessary, observe where x approaches as y approaches $\pm\infty$.
5. Find points of local maximum and local minimum
   Put $f'(x) = 0$ and find points of local maximum and minimum.

4.2 Important Results

1. If $f(x) \geq 0$ for all $x \in [a, b]$, then Area bounded by the curve $y = f(x)$, X-axis and the lines $x = a$ and $x = b$ is given by

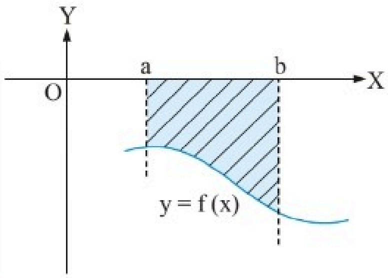
$$A = \int_a^b f(x)dx$$



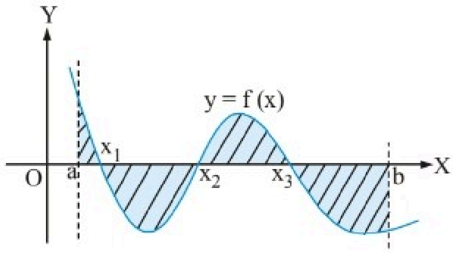
Note : The whole of the curve in the internal $[a, b]$ lies above X-axis.

2. If $f(x) \leq 0$ for all $x \in [a, b]$, then Area bounded by a curve $y = f(x)$. X-axis and the lines $x = a$ and $x = b$ is given by :

$$\text{Area} = \left|\int_a^b f(x)\,dx\right|$$



3. If the curve crosses X-axis one or more times in $[a, b]$, then the area bounded by the curve $y = f(x)$, X-axis and the lines $x = a$ and $x = b$ is calculated by considering the portions of the graph lying above X-axis and below X-axis separately. To calculate the area of the regions lying above X-axis, use result-1 and for the regions lying below X-axis, use result-2.

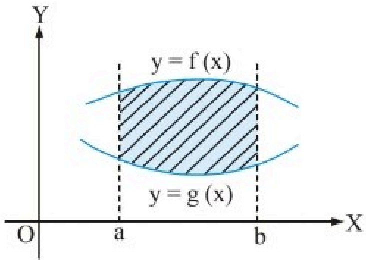


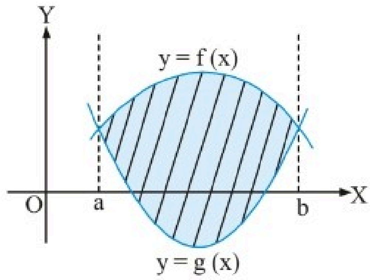
In the figure, the curve crosses X-axis at $x = x_1, x_2, x_3$.
Shaded area is given as follows :

$$A = \int_a^{x_1} f(x)\,dx + \left|\int_{x_1}^{x_2} f(x)\,dx\right| + \int_{x_2}^{x_3} f(x)\,dx + \left|\int_{x_3}^{b} f(x)\,dx\right|$$

4. Area bounded by two curves, $y = f(x)$ and $y = g(x)$, from above and below is given by :

$$\text{shaded area} = \int_a^b [f(x) - g(x)]\,dx$$





Note : The area is bounded from above by $y = f(x)$ and from below by $y = g(x)$.
The shaded area may be above or below X-axis.

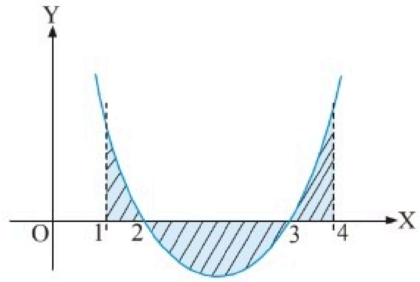
Illustration - 25 *The area bounded by the curve $y = x^2 - 5x + 6$, X-axis and the lines $x = 1$ and 4 is :*

(A) $\frac{9}{6}$ (B) $\frac{10}{6}$ (C) $\frac{11}{6}$ (D) *None of these*

SOLUTION : (C)

For $y = 0$, we get $x^2 + 5x + 6 = 0$
$\Rightarrow x = 2, 3$
Hence the curve crosses X-axis at $x = 2, 3$ in the interval $[1, 4]$.

$$\text{Bounded Area} = \int_1^2 y\,dx + \left|\int_2^3 y\,dx\right| + \int_3^4 y\,dx$$



$$\Rightarrow \quad A = \int_1^2 (x^2 - 5x + 6)\,dx + \left| \int_2^3 (x^2 - 5x + 6)\,dx \right| + \int_3^4 (x^2 - 5x + 6)\,dx$$

$$A_1 = \frac{2^3 - 1^3}{3} - 5\left(\frac{2^2 - 1^2}{2}\right) + 6\,(2-1) = \frac{5}{6}$$

$$A_2 = \frac{3^3 - 2^3}{3} - 5\left(\frac{3^2 - 2^2}{2}\right) + 6\,(3-2) = -\frac{1}{6}$$

$$A = \frac{4^3 - 3^3}{3} - 5\left(\frac{4^2 - 3^2}{2}\right) + 6\,(4-3) = \frac{5}{6}$$

$$\Rightarrow A = \frac{5}{6} + \left| -\frac{1}{6} \right| + \frac{5}{6} = \frac{11}{6} \text{ sq. units.}$$

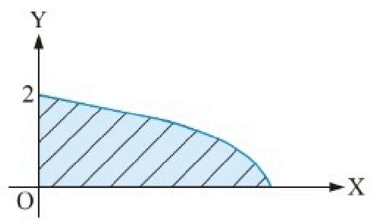
Illustration - 26 *The area bounded by the curve : $y = \sqrt{4-x}$, X-axis and Y-axis.*

(A) $\frac{8}{3}$ **(B)** $\frac{16}{3}$ **(C)** $\frac{32}{3}$ **(D)** *None of these*

SOLUTION : (B)

Trace the curve $y = \sqrt{4-x}$.

1. Put $y = 0$ in the given curve to get $x = 4$ as the point of intersecton with X-axis.
   Put $x = 0$ in the given curve to get $y = 2$ as the point of intersection with Y-axis.
2. For the curve, $y = \sqrt{4-x}$, $4 - x \geq 0$
   $\Rightarrow$ $x \leq 4$
   $\Rightarrow$ curve lies only to the left of $x = 4$ line.
3. As any y is positive, curve is above X-axis.



Using step 1 to 3, we can draw the rough sketch of $y = \sqrt{4-x}.$

In figure,

Bounded area =

$$\int_0^4 \sqrt{4-x}\,dx = \left| \frac{-2}{3}(4-x)\sqrt{4-x} \right|_0^4 = \frac{16}{3} \text{ sq. units.}$$

Illustration - 27 *The area bounded by the curves $y = x^2$ and $x^2 + y^2 = 2$ above X-axis is :*

(A) $\frac{1}{6}+\frac{\pi}{4}$ (B) $\frac{2}{3}+\frac{\pi}{2}$ (C) $\frac{\pi}{4}+\frac{1}{6}$ (D) $\frac{1}{3}+\frac{\pi}{2}$

SOLUTION : (D)

Let us first find the points of intersection of curves.

Solving $y = x^2$ and $x^2 + y^2 = 2$ simultaneously,

we get :

$$x^2 + x^4 = 2$$

$\Rightarrow (x^2 - 1)(x^2 + 2) = 0$

$\Rightarrow x^2 = 1$ and $x^2 = -2$ [reject]

$\Rightarrow x = \pm 1$

$\Rightarrow A = (-1, 0)$ and $B = (1, 1)$

$$\text{Shaded Area} = \int_{-1}^{+1}\left(\sqrt{2-x^2} - x^2\right)dx$$

$$= \int_{-1}^{+1}\sqrt{2^x - x^2}\,dx - \int_{-1}^{+1} x^2\,dx$$

$$= 2\int_0^1 \sqrt{2-x^2}\,dx - 2\int_0^1 x^2\,dx$$

$$= 2\left|\frac{x}{2}\sqrt{2-x^2} + \frac{2}{2}\sin^{-1}\frac{x}{\sqrt{2}}\right|_0^1 - 2\left(\frac{1}{3}\right)$$

$$= 2\left(\frac{1}{2}+\frac{\pi}{4}\right) - \frac{2}{3} = \frac{1}{3}+\frac{\pi}{2} \text{ sq. units.}$$

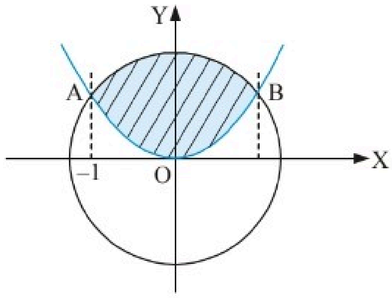


Illustration - 28 *The area bounded by $y = x^2 - 4$ and $x + y = 2$ is :*

(A) $\frac{75}{6}$ (B) $\frac{100}{6}$ (C) $\frac{125}{6}$ (D) $\frac{150}{6}$

SOLUTION : (C)

After drawing the figure, let us find the points of intersection of

$y = x^2 - 4$ and $x + y = 2$.

$\Rightarrow x + x^2 - 4 = 2 \Rightarrow x^2 + x - 6 = 0$

$\Rightarrow (x + 3)(x - 2) = 0$

$\Rightarrow x = -3, 2$

$\Rightarrow A \equiv (-3, 5)$ and $B \equiv (2, 0)$

$$\text{Shaded area, } = \int_{-3}^{2}\left[(2-x)-(x^2-4)\right]dx$$

$$= \int_{-3}^{2}(2-x)\,dx - \int_{-3}^{2}(x^2-4)\,dx$$

$$= \left|2x - \frac{x^2}{2}\right|_{-3}^{2} - \left|\frac{x^3}{3} - 4x\right|_{-3}^{2}$$

$$= 2\times 5 - \frac{1}{2}(4-9) - \frac{1}{3}(8+27) + 4(5) = \frac{125}{6}$$

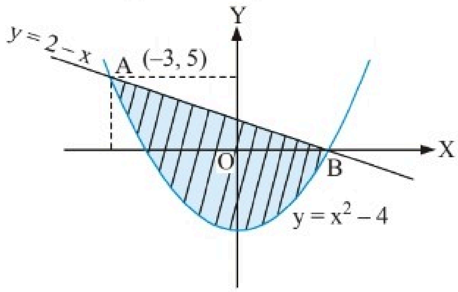


Illustration - 29 *The area bounded by the circle $x^2 + y^2 = a^2$ is :*

(A) $\frac{\pi a^2}{4}$ (B) $\frac{\pi a^2}{2}$ (C) πa^2 (D) $2\pi a^2$

SOLUTION : (C)

$x^2 + y^2 = a^2 \quad \Rightarrow \quad y = \pm\sqrt{a^2 - x^2}$

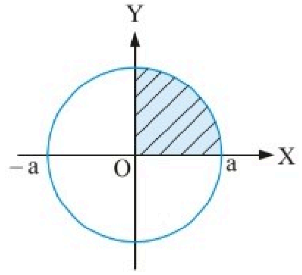
Equation of semicircle above X-axis is $y = +\sqrt{a^2 - x^2}$

Area of circle = 4 (shaded area)

$$= 4\int_0^a \sqrt{a^2 - x^2}\, dx$$

$$= 4\left|\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right|_0^a$$

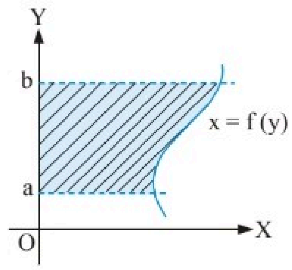
$$= 4\frac{a^2}{2}\left(\frac{\pi}{2}\right) = \pi a^2$$



4.3 Important Results (Contd.....)

5. If $f(y) \geq 0$ for all $y \in [a, b]$, then the Area bounded by a curve $x = f(y)$, Y-axis and the lines $y = a$ and $y = b$ is given by

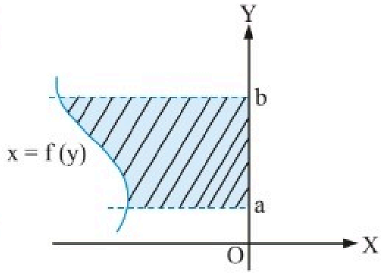
$$\text{Area} = \int_a^b f(y)\, dy$$



Note : The whole of the curve in the interval $[a, b]$ lies on right of *Y*-axis.

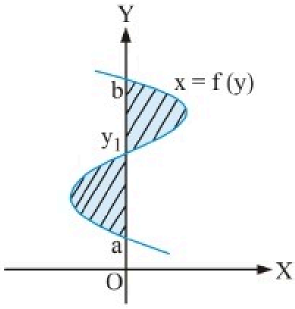
6. If $f(y) \leq 0$ for all $y \in [a, b]$, then the Area bounded by a curve $x = f(y)$, Y-axis and the lines $y = a$ and $y = b$ is given by

$$\text{Area} = \left|\int_a^b f(y)\, dy\right|$$



Note : The whole of the curve in the interval $[a, b]$ lies on left of Y-axis.

7. If the curve crosses Y-axis one or more times in $[a, b]$, then the area bounded by the curve $x = f(y)$, Y-axis and the lines $y = a$ and $y = b$ is calculated by considering the portions of the graph lying on the right side and the left side of the Y-axis separately. To calculate the area of the regions lying on right-hand side of the Y-axis, use result-5 and for the regions lying on left-hand side, use result - 6.



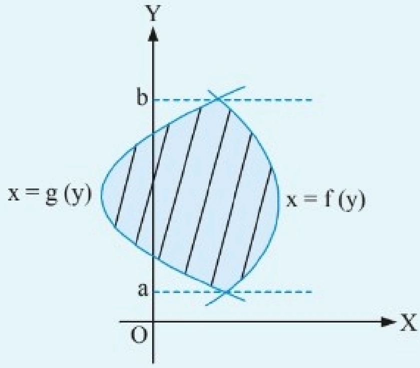
In the figure, the curve crosses Y-axis at $y = y_1$.

Shaded area is given as follows : $A = \left|\int_a^{y_1} f(x)\,dy\right| + \int_{y_1}^{b} f(y)\,dy$

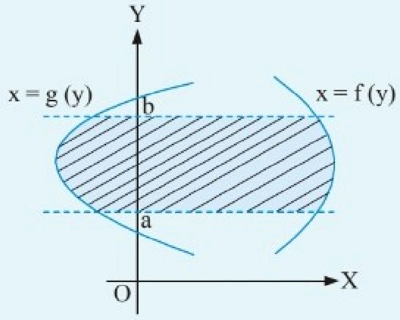
8. Area bounded by two curves, $x = f(y)$ and $x = g(y)$, from right and left respectively, is given by :

Shaded area = $\int_a^b [f(y) - g(y)]\,dy$

Note : The area is bounded from right by $x = f(y)$ and from left by $x = g(y)$.



The shaded area may be on right or left side of the Y-axis.



9. If the equations of the curves are expressed in parametric form, then the area bounded can not be found by direct application of the result 1 to 8.
Let the two curves in parametric form are

$x = f(t)$ ...(i) and $y = g(t)$ ...(ii)

To find the bounded area by curves, try to eliminate parameter t in equations (i) and (ii) to express y in terms of x (or x in terms of y). If it is possible to eliminate t, then the required area can be obtained by using the results 1 to 8.
If it is not possible to eliminate t, then the required area can be obtained by using the following formula :

Area $= \int_a^b y\,dx = \int_a^b y\frac{dx}{dt}\,dt = \int_{t_1}^{t_2} g(t)f'(t)\,dt$ where t_1 and t_2 are given by $f(t_1) = a$ and $f(t_2) = b$.

Illustration - 30 *The area bounded by the curves $x^2 + y^2 = 4a^2$ and $y^2 = 3ax$ is :*

(A) $\left(\frac{1}{\sqrt{3}}+\frac{4\pi}{3}\right)a^2$ (B) $\left(\frac{1}{2\sqrt{3}}+\frac{2\pi}{3}\right)a^2$ (C) $\left(\frac{1}{\sqrt{3}}+\frac{2\pi}{3}\right)a^2$ (D) $\left(\frac{1}{2\sqrt{3}}+\frac{4\pi}{3}\right)a^2$

SOLUTION : (A)

The points of intersection A & B can be calculated.
by solving $x^2 + y^2 = 4a^2$ and $y^2 = 3ax$.

$\Rightarrow \left(\frac{y^2}{3a}\right)^2 + y^2 = 4a^2$

$\Rightarrow y^4 + 9a^2y^2 - 36a^4 = 0$

$\Rightarrow (y^2 - 3a^2)(y^2 + 12a^2) = 0$

$\Rightarrow y^2 = 3a^2$ or $y^2 = -12a^2$ (reject)

$\Rightarrow y^2 = 3a^2 \Rightarrow y = \pm\sqrt{3}a$

The equation of right half of

$x^2 + y^2 = 4a^2$ is $x = \sqrt{4a^2 - y^2}$

Shaded area $= \int_{-\sqrt{3}a}^{\sqrt{3}a}\left(\sqrt{4a^2-y^2} - \frac{y^2}{3a}\right)dy$

$= 2\int_0^{\sqrt{3}a}\left(\sqrt{4a^2-y^2} - \frac{y^2}{3a}\right)dy$

[using property - 8]

$= 2\left|\frac{y}{2}\sqrt{4a^2-y^2} + \frac{4a^2}{2}\sin^{-1}\frac{y}{2a}\right|_0^{\sqrt{3}a} - \frac{2}{3a}\left|\frac{y^3}{3}\right|_0^{\sqrt{3}a}$

$= \sqrt{3a^2} + 4a^2\frac{\pi}{3} - \frac{2}{9a}3\sqrt{3}a^3$

$= \left(\frac{1}{\sqrt{3}} + \frac{4\pi}{3}\right)a^2$

Alternative Method :

shaded area $= 2 \times$ (area above X-axis)

x-coordinate of $A = \frac{y^2}{3a} = \frac{3a^2}{3a} = a$

The given curves area

$y = \pm\sqrt{3ax}$ and $y = \pm\sqrt{4a^2 - x^2}$

The above the X-axis, the equations of the parabola and the circle are $\sqrt{3ax}$ and $y = \sqrt{4a^2 - x^2}$ respectively.

$\Rightarrow$ Shaded area

$$= 2\left[\int_0^a \sqrt{3ax}\,dx + \int_a^{2a} \sqrt{4a^2 - x^2}\,dx\right]$$

Solve it yourself to get the answer.

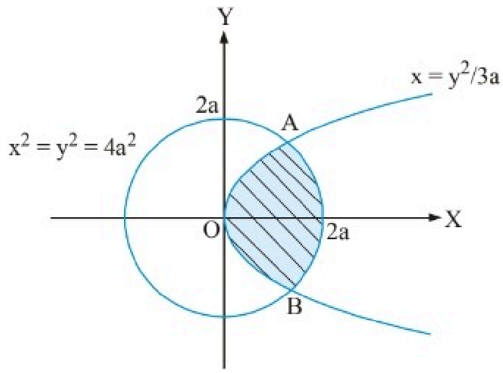


Illustration - 31 *The area bounded by the curves : $y^2 = 4a\,(x + a)$ and $y^2 = 4b\,(b - x)$ is :*

(A) $\frac{2}{3}(a+b)\sqrt{4ab}$ (B) $\frac{4}{3}(a+b)\sqrt{4ab}$ (C) $\frac{8}{3}(a+b)\sqrt{4ab}$ (D) *None of these*

SOLUTION : (B)

The two curves are :

$y^2 = 4a\,(x + a)$ ... (i)

and $y^2 = 4b\,(b - x)$ ... (ii)

Solving $y^2 = 4a\,(x + a)$ and $y^2 = 4b\,(b - x)$ simultaneously,

we get the coordinates of A and B.

Replacing values of x from (ii) and (i), we get :

$$y^2 = 4a\left(b - \frac{y^2}{4b} + a\right)$$

$\Rightarrow\ y = \pm\sqrt{4ab}$ and $x = b - a$.

$\Rightarrow A \equiv (b - a, \sqrt{4ab})$ and $B \equiv (b - a, -\sqrt{4ab})$

$$\text{shaded area} = \int_{-\sqrt{4ab}}^{\sqrt{4ab}} \left[\left(b - \frac{y^2}{4b}\right) - \left(\frac{y^2}{4a} - a\right)\right] dy$$

$$\Rightarrow\ A = 2\,(a+b)\sqrt{4ab} - \int_0^{\sqrt{4ab}} \left(\frac{y^2}{2b} + \frac{y^2}{2a}\right) dy$$

[using property - 8]

$$\Rightarrow A = 2\,(a+b)\,\sqrt{4ab} - \frac{1}{2}\left[\frac{4ab\sqrt{4ab}}{3b} + \frac{4ab\sqrt{4ab}}{3a}\right]$$

$$\Rightarrow\ A = \frac{4}{3}\,(a+b)\,\sqrt{4ab}$$

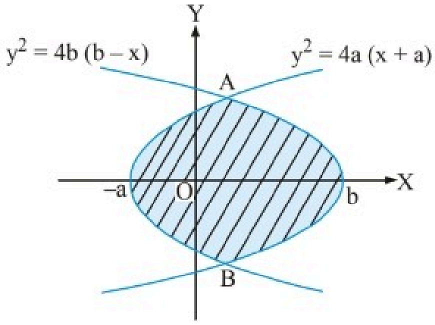


Illustration - 32 *The area bounded by the hyperbola : $x^2 - y^2 = a^2$ and the line $x = 2a$ is :*

(A) $\sqrt{3}\,a^2 - a^2 \log\left(2+\sqrt{3}\right)$ (B) $2\sqrt{3}\,a^2 - a^2 \log\left(2+\sqrt{3}\right)$

(C) $\sqrt{3}\,a^2 - a^2 \log\left(2-\sqrt{3}\right)$ (D) $2\sqrt{3}\,a^2 - a^2 \log\left(2-\sqrt{3}\right)$

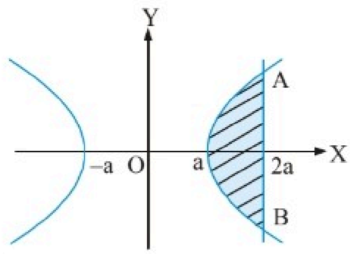
SOLUTION : (B)

Shaded area $= 2 \times$ (Area of the portion above X- axis)

The equation of the curve above x-axis is :

$$y = \sqrt{x^2 - a^2}$$

$$\Rightarrow \text{ required area } (A) = 2\int_a^{2a} \sqrt{x^2 - a^2}\,dx$$



$$\Rightarrow A = 2\left|\frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right|\right|_a^{2a}$$

$$\Rightarrow A = 2\sqrt{3}\,a^2 - a^2 \log\left|(2a + \sqrt{3a})\right| + a^2 \log a$$

$$\Rightarrow A = 2\sqrt{3}\,a^2 - a^2 \log(2 + \sqrt{3}).$$

Alternative Method :

$$\text{Area } (A) = \int_{y_B}^{y_A} \left(2a - \sqrt{a^2 + y^2}\right) dy$$

$$\Rightarrow A = \int_{-\sqrt{3}a}^{\sqrt{3}a} \left(2a - \sqrt{a^2 + y^2}\right) dy$$

Solve it yourself to confirm.

Illustration - 33 *The area bounded by the curves : $x^2 + y^2 = 25$, $4y = |4 - x^2|$ and $x = 0$ in the first quadrant is :*

(A) $2 + \dfrac{25}{2}\sin^{-1}\left(\dfrac{4}{5}\right)$ (B) $2 + \dfrac{25}{4}\sin^{-1}\left(\dfrac{4}{5}\right)$

(C) $1 + \dfrac{25}{2}\sin^{-1}\left(\dfrac{4}{5}\right)$ (D) $1 + \dfrac{25}{4}\sin^{-1}\left(\dfrac{4}{5}\right)$

SOLUTION : (A)

First of all find the coordinates of points of intersec -tion A by solving the equations of two given curves :

$$\Rightarrow x^2 + y^2 = 25 \quad \text{and} \quad 4y = |4 - x^2|$$

$$\Rightarrow x^2 + \frac{(4 - x^2)^2}{16} = 25$$

$$\Rightarrow (x^2 - 4)^2 + 16x^2 = 400.$$

$$\Rightarrow (x^2 + 4)^2 = 400 \quad \Rightarrow \quad x^2 = 16$$

$$\Rightarrow x = \pm 4 \quad \Rightarrow \quad y = \frac{|4 - x^2|}{4} = 3$$

$\Rightarrow$ Coordinates of point are $A \equiv (4, 3)$

$$\text{Shaded area} = \int_0^4 \left[\sqrt{25 - x^2} - \frac{|4 - x^2|}{4}\right] dx$$

$$\Rightarrow A = \int_0^4 \sqrt{25 - x^2}\,dx - \frac{1}{4}\int_0^4 |4 - x^2|\,dx \quad \ldots \text{(i)}$$

Let $I = \frac{1}{4}\left|4 - x^2\right| dx$

$\Rightarrow \; I = \frac{1}{4}\int_0^2 (4 - x^2)\, dx + \frac{1}{4}\int_2^4 (x^2 - 4)\, dx$

$\Rightarrow \; I = \frac{1}{4}\left(8 - \frac{8}{3}\right) - \frac{1}{4}\left(\frac{56}{3} - 8\right)$

$\Rightarrow \quad I = 4$

On substituting the value of I in (i), we get :

$$A = \int_0^4 \sqrt{25 - x^2}\, dx - 4$$

$\Rightarrow A = \left|\frac{x}{2}\sqrt{25 - x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right|_0^4 - 4$

$\Rightarrow A = 6 + \frac{25}{2}\sin^{-1}\frac{4}{5} - 4 = 2 + \frac{25}{2}\sin^{-1}\frac{4}{5}$

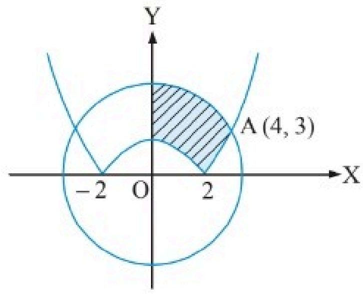


Illustration - 34 *The area enclosed by the loop in the curve : $4y^2 = 4x^2 - x^3$ is :*

(A) $\frac{32}{15}$ (B) $\frac{64}{15}$ (C) $\frac{128}{15}$ (D) $\frac{256}{15}$

SOLUTION : (C)

The given curve is : $4y^2 = 4x^2 - x^3$

To draw the rough sketch of the given curve, consider the following steps :

(i) On replacing y by $-y$, there is no change in function. It means the graph is symmetric about Y-axis.

(ii) For $x = 4, y = 0$ and for $x = 0, y = 0$.

(iii) In the given curve, LHS is positive for all values of y.

$\Rightarrow \quad \text{RHS} \geq 0 \quad \Rightarrow \quad x^2(1 - x/4) \geq 0$

$\Rightarrow \quad x \leq 4.$

Hence the curve lies to the left of $x = 4$.

(iv) As $x \to -\infty, y \to \pm\infty$

(v) Points of maximum/minimum :

$$8y\frac{dy}{dx} = 8x - 3x^2$$

$\frac{dy}{dx} = 0 \quad \Rightarrow \quad x = 0, \frac{8}{3}$

At $x = 0$, derivative is not defined.

By checking for $\frac{d^2y}{dx^2}$, $x = \frac{8}{3}$ is a point of local maximum (above X-axis).

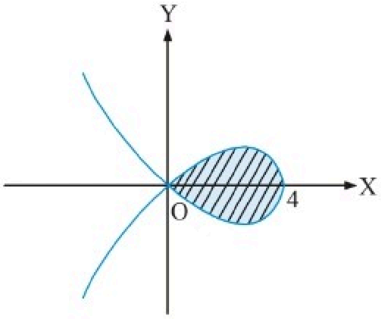
From graph,

Shaded area $(A) = 2 \times$ (area of portion above X-axis)

$\Rightarrow \; A = 2\int_0^4 \frac{x}{2}\sqrt{4 - x}\, dx = \int_0^4 x\sqrt{4 - x}\, dx$

$\Rightarrow \; A = \int_0^4 (4 - x)\sqrt{4 - (4 - x)}\, dx$

[using property - 4]



$$\Rightarrow A = \int_0^4 (4-x)\sqrt{x}\, dx$$

$$\Rightarrow A = 4\left|\frac{2}{3}x\sqrt{x}\right|_0^4 - \left|\frac{2}{5}x^2\sqrt{x}\right|_0^4$$

$$\Rightarrow A = \frac{128}{15} \text{ sq. units.}$$

Illustration - 35 *The area bounded by the parabola $y = x^2$, X-axis and the tangent to the parabola at* (1, 1) *is* :

(A) $\frac{1}{4}$ (B) $\frac{1}{6}$ (C) $\frac{1}{9}$ (D) $\frac{1}{12}$

SOLUTION : (D)

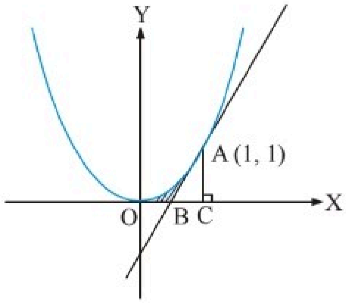
The given curve is $y = x^2$. Equation of tangent at $A = (1, 1)$ is :

$$y - 1 = \left.\frac{dy}{dx}\right]_{x=1} \cdot \quad (x-1) \qquad \text{[using : } y - y_1 = m(x - x_1)\text{]}$$

$$\Rightarrow \quad y - 1 = 2(x-1) \quad \Rightarrow \quad y = 2x - 1 \qquad \text{... (i)}$$

The point of intersection of (i) with X-axis is $B \equiv (1/2, 0)$.

Shaded area = area $(OACO)$ – area (ABC)



$$\Rightarrow \quad \text{area} = \int_0^1 x^2 dx - \int_{1/2}^1 (2x-1)\, dx$$

$$\Rightarrow \quad \text{area} = \frac{1}{3}\left[1 - \frac{1}{4} - (1 - 1/2)\right]$$

$$\Rightarrow \quad \text{area} = \frac{1}{12}$$

Illustration - 36 *The area between the curves $y = 2x^4 - x^2$, the x-axis and the ordinates of two minima of the curve is :*

(A) $\frac{7}{240}$ (B) $\frac{7}{120}$ (C) $\frac{7}{60}$ (D) *None of these*

SOLUTION : (B)

Using the curve tracing steps , draw the rough sketch of the functions $y = 2x^4 - x^2$.

Following are the properties of the curve which can be used to draw its rough sketch.

(i) The curve is symmetrical about y-axis.

(ii) Point of intersection with x-axis are $x = 0, x = \pm\frac{1}{\sqrt{2}}$. Only point of intersection with y-axis is $y = 0$.

(iii) For $x \in \left(-\infty, -\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, \infty\right)$, $y > 0$ i.e. curve lies above x axis and in the other intervals it lies below x-axis.

(iv) Put $\frac{dy}{dx} = 0$ to get $x = \pm 1/2$ as the points of local minimum.

On plotting the above information on graph, we get the rough sketch of the graph. The shaded area in the graph is the required area

$$\text{Required Area} = 2\left|\int_0^{1/2}\left(2x^4 - x^2\right)dx\right|$$

$$= 2\left|\left[\frac{2x^5}{5} - \frac{x^3}{3}\right]_0^{1/2}\right| = \frac{7}{120}$$