Qu: Suppose a differentiable f^n $f(x)$ satisfies the identity $f(x+y) = f(x) + f(y) + xy^2 + x^2y$ for all real x & y

$\lim_{x \to 0} \frac{f(x)}{x} = 1$, then $f'(3)$ is equal to ______

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Sol: - $$f(x+y) = f(x) + f(y) + xy^2 + x^2y \quad \text{—— (i)}$$

$$f'(x+y) = f'(x) + 0 + y^2 + y(2x) \quad \text{—— (ii)}$$

Putting $x=0$ and $y=x$

$$f'(x) = f'(0) + x^2 \quad \text{—— (iii)}$$

Putting $x=y=0$ in eq^n (i)

$$f(0) = 2f(0) \Rightarrow f(0) = 0$$

$$\lim_{x \to 0} \frac{f(x)}{x} = 1$$

This is in $\frac{0}{0}$ form, so we can apply L'hospital rule

$$\lim_{x \to 0} \frac{f'(x)}{1} = 1$$

$$\Rightarrow \boxed{f'(0) = 1}$$

Putting value of $f'(0)$ at eq^n (iii).

we get $f'(x) = 1 + x^2 \Rightarrow \boxed{f'(3) = 1 + 3^2 = 10}$ Ans

Qu: Which of the following f^n have a finite no. of points of discontinuity in R?

[A] $\tan x$

[B] $x[x]$

[C] $|x|/x$

[D] $\sin[\pi x]$

Sol:. $f(x) = \tan x$ is discontinous when $x = \frac{(2n+1)\pi}{2}$, $n \in Z$

$f(x) = x[x]$ is discontinous when $x = k$, $k \in Z$

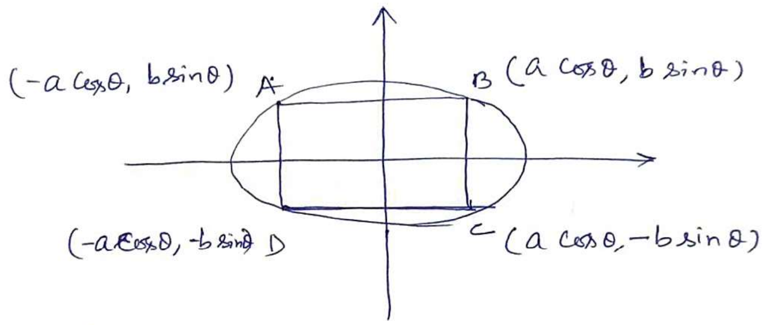
$f(x) = \sin[n\pi x]$ is discontinous when $n\pi x = k$, $k \in Z$

Thus all the above f^ns have an ∞ no. of points of discontinuity.

But if $\frac{|x|}{x}$ is discontinous only when $x=0$. Ans [C] $\frac{|x|}{x}$

Qu: Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is:

Sol=



Area of rectangle ABCD, $= 2a\cos\theta(2b\sin\theta) = 2ab\sin 2\theta$

Let $f(\theta) = 2ab\sin 2\theta$

$f'(\theta) = 4ab\cos 2\theta$

for $f(\theta) \Rightarrow$ Maxima value $f'(\theta) = 0$

$2\theta = \frac{(2n+1)\pi}{2}$ where $n = 0, 1, 2, \ldots$

again $f''(\theta) = -8ab\sin 2\theta$

taking $(n = 0)$ $f''(\frac{\pi}{4}) = -8ab \Rightarrow -ve.$

Thus for $\frac{\pi}{4}$ $f(\theta) =$ is Max^{m}.

$f(\frac{\pi}{4}) = \underline{\underline{2ab}}$ $\underline{\underline{\text{Ans}}}$