Que:

If the function $f(x) = \begin{cases} \frac{1}{x}\log_e\left(\dfrac{1+\frac{x}{a}}{1-\frac{x}{b}}\right), & x<0 \\ k, & x=0 \\ \dfrac{\cos^2 x - \sin^2 x - 1}{(\sqrt{x^2+1})-(1)}, & x>0 \end{cases}$ is continuous at $x=0$,

then $\frac{1}{a}+\frac{1}{b}+\frac{4}{k}$ is equal to?

Sol:- If $f(x)$ is continous at $x=0$, RHS = LHS = f(0)

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\cos^2 x - \sin^2 x - 1}{(\sqrt{x^2+1})-(1)}$$

or $$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\cos 2x - 1}{\{\sqrt{x^2+1}-1\}} \cdot \frac{\{\sqrt{x^2+1}+1\}}{\{\sqrt{x^2+1}+1\}} = \lim_{x\to 0^+} \frac{-2\sin^2 x}{x^2}\cdot(\sqrt{x^2+1}+1)$$

$$\lim_{x\to 0^+} f(x) = -4$$

and $$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{1}{x}\left(\log_e\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right)\right) = \lim_{x\to 0^-} \frac{\ln(1+\frac{x}{a})}{(\frac{x}{a})\cdot a} + \frac{\ln(1-\frac{x}{b})}{(-\frac{x}{b})\cdot b}$$

$$= \frac{1}{a}+\frac{1}{b}$$

So, $$\frac{1}{a}+\frac{1}{b} = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = -4 = \lim_{x\to 0} f(x) = k$$

$$\Rightarrow \boxed{\frac{1}{a}+\frac{1}{b}+\frac{4}{k} = -4-1 = -5}$$

Ans = -5

Qu: Let $f: [-1,1] \to R$ be defined as $f(x) = ax^2+bx+c$ for all $x \in [-1,1]$ where $a, b, c \in R$ such that $f(-1) = 2$, $f'(-1) = 1$ for $x \in (-1,1)$ the maximum value of $f''(x)$ is $\frac{1}{2}$. If $f(x) \le \alpha$, $x \in [-1,1]$, then the least value of $\alpha$ is equal to ______

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Sol: $f(x) = ax^2+bx+c$

$f'(x) = 2ax+b$

$f''(x) = 2a$

given that, $f''(-1) = \frac{1}{2} \Rightarrow 2a = \frac{1}{2} \Rightarrow \boxed{a = \frac{1}{4}}$

again $f'(-1) = 1 \Rightarrow b - 2a = 1 \Rightarrow \boxed{b = \frac{3}{2}}$ $\boxed{b = \frac{3}{2}}$

again $f(-1) = a - b + c = 2 \Rightarrow \boxed{c = \frac{13}{4}}$

Now, $f(x) = \frac{1}{4}(x^2+6x+13)$, $x \in [-1, 1]$

$f'(x) = \frac{1}{4}(2x+6) = 0 \Rightarrow x = -3 \notin [-1,1]$

$f(1) = 5$ ; $f(-1) = 2$

$\Rightarrow f(x) \le 5$ So $\boxed{\alpha_{min^m} = 5}$ Ans

Qu: The number of points at which the $f^n$ $f(x) = |2x+1| - 3|x+2| + |x^2+x-2|$, $x \in R$ is not differentiable is ______

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Sol:- $f(x) = |2x+1| - 3|x+2| + |x^2+x-2|$, $x \in R$

$$f(x) = \begin{cases} x^2-7, & x > 1 \\ -x^2-2x-3, & -\frac{1}{2} < x < 1 \\ -x^2-6x-5, & -2 < x < -\frac{1}{2} \\ x^2+2x+3, & x < -2 \end{cases}$$

$$f'(x) = \begin{cases} 2x, & x > 1 \\ 2x-3, & -\frac{1}{2} < x < 1 \\ -2x-6, & -2 < x < -\frac{1}{2} \\ 2x+2, & x < -2 \end{cases}$$

Checking at $x = 1$, $-2$ and $-\frac{1}{2}$

Thus, Non. differentiable at $x = 1$ an $-\frac{1}{2}$ Ans