Paragraph for Questions 1–3: Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real valued differentiable function $y = f(x)$.

If $x \in (-2, 2)$, the equation implicitly defines a unique real valued differentiable function $y = g(x)$ satisfying $g(0) = 0$.

[IIT-JEE 2008]

1. If $f(-10\sqrt{2}) = 2\sqrt{2}$, then $f''(-10\sqrt{2}) =$

(A) $\frac{4\sqrt{2}}{7^3 3^2}$ (B) $-\frac{4\sqrt{2}}{7^3 3^2}$

(C) $\frac{4\sqrt{2}}{7^3 3}$ (D) $-\frac{4\sqrt{2}}{7^3 3}$

Solution: We have

$$y^3 - 3y + x = 0$$

Differentiate both sides, we get

$$3y^2 y' - 3y' + 1 = 0 \quad (1)$$

Put $y = 2\sqrt{2},\quad x = -10\sqrt{2}$. Then

$$y'(-10\sqrt{2}) = \frac{-1}{21}$$

Differentiate equation (1), we get

$$3y^2 y'' + 6y(y')^2 - 3y'' = 0$$

Put $y = 2\sqrt{2}, x = -10\sqrt{2}, y' = \frac{-1}{21}$. Then

$$y''(-10\sqrt{2}) = -\frac{4\sqrt{2}}{7^3 \cdot 3^2}$$

Hence, the correct answer is option (B).

2. The area of the region bounded by the curves $y = f(x)$, the x-axis, and the lines $x = a$ and $x = b$, where $-\infty < a < b < -2$, is

(A) $\int_a^b \frac{x}{3((f(x))^2 - 1)} dx + bf(b) - af(a)$

(B) $-\int_a^b \frac{x}{3((f(x))^2 - 1)} dx + bf(b) - af(a)$

(C) $\int_a^b \frac{x}{3((f(x))^2 - 1)} dx - bf(b) + af(a)$

(D) $-\int_a^b \frac{x}{3((f(x))^2-1)}dx - bf(b) + af(a)$

Solution:

$$\text{Required area} = \int_a^b f(x)dx$$
$$= [xf(x)]_a^b - \int_a^b xf'(x)dx \qquad \text{(By parts)}$$
$$= bf(b) - af(a) + \int_a^b \frac{xdx}{3[f(x)^2-1]}$$

Hence, the correct answer is option (A).

3. $\int_{-1}^{1} g'(x)dx =$

(A) $2g(-1)$ **(B)** 0 **(C)** $-2g(1)$ **(D)** $2g(1)$

Solution:

$$y' = \frac{1}{3[1-(f(x))^2]}$$

Clearly $f(x)$ is an odd function, then $g'(x)$ is an even function, so

$$\int_{-1}^{1} g'(x)dx = 2\int_0^1 g'(x)dx$$
$$= 2[g(x)]_0^1$$
$$= 2[g(1) - g(0)]$$
$$= 2g(1) \qquad (\text{As } g(0) = 0)$$

Hence, the correct answer is option (D).

4. The area of the region between the curves $y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the lines $x = 0$ and $x = \frac{\pi}{4}$ is

(A) $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}}dt$ **(B)** $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$

(C) $\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$ **(D)** $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}}dt$

[IIT-JEE 2008]

Solution: Since, both curves lie above x-axis in $x \in \left(0, \frac{\pi}{4}\right)$.

Therefore, area bounded between the curve is

$$\int_0^{\frac{\pi}{4}} \left(\sqrt{\frac{1+\sin x}{\cos x}} - \sqrt{\frac{1-\sin x}{\cos x}}\right)dx$$
$$= \int_0^{\frac{\pi}{4}} \left(\sqrt{\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}} - \sqrt{\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}}\right)dx$$
$$= \int_0^{\frac{\pi}{4}} \left(\frac{2\tan\frac{x}{2}}{\sqrt{1-\tan^2\frac{x}{2}}}\right)dx$$

Put $\tan\frac{x}{2} = t$. Then

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$
$$\Rightarrow dx = \frac{2dt}{1+t^2}$$
$$\Rightarrow \int_0^{\sqrt{2}-1} \left(\frac{4t}{(1+t^2)\sqrt{1-t^2}}\right)dt$$

Hence, the correct answer is option (B).

5. Area of the region bounded by the curve $y = e^x$ and lines $x = 0$ and $y = e$ is

(A) $e - 1$ **(B)** $\int_1^e \ln(e+1-y)dy$

(C) $e - \int_0^1 e^x dx$ **(D)** $\int_1^e \ln y\, dy$

[IIT-JEE 2009]

Solution: See Fig. 24.41.

$$\text{Required area} = \int_1^e \ln y\, dy$$
$$= (y\ln y - y)_1^e = (e-e) - \{-1\} = 1$$

Also,

$$\int_1^e \ln y\, dy = \int_1^e \ln(e+1-y)dy$$

Further the required area can be written as

$$e \times 1 - \int_0^1 e^x dx$$

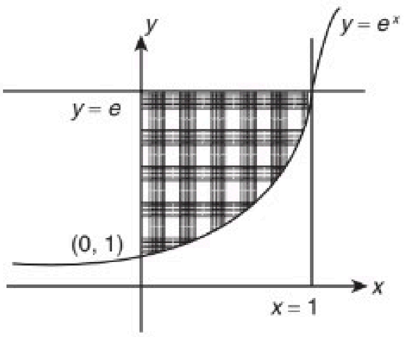

Figure 24.41

Hence, the correct answers are options (B), (C) and (D).

Paragraph for questions 6–8: Consider the polynomial $f(x) = 1 + 2x + 3x^2 + 4x^3$. Let s be the sum of all distinct real roots of $f(x)$ and let $t = |s|$.

[IIT-JEE 2010]

6. The real number s lies in the interval

(A) $\left(-\frac{1}{4}, 0\right)$ **(B)** $\left(-11, -\frac{3}{4}\right)$

(C) $\left(-\frac{3}{4}, -\frac{1}{2}\right)$ **(D)** $\left(0, \frac{1}{4}\right)$

Solution: Since,

$$f\left(-\frac{1}{2}\right)\cdot f\left(-\frac{3}{4}\right) < 0$$

$$\Rightarrow S \text{ lie in } \left(-\frac{3}{4}, -\frac{1}{2}\right)$$

Hence, the correct answer is option (C).

7. The area bounded by the curve $y = f(x)$ and the lines $x = 0$, $y = 0$ and $x = t$, lies in the interval

(A) $\left(\frac{3}{4}, 3\right)$ **(B)** $\left(\frac{21}{64}, \frac{11}{16}\right)$

(C) $(9, 10)$ **(D)** $\left(0, \frac{21}{64}\right)$

Solution:

$$-\frac{3}{4} < s < -\frac{1}{2}$$

$$\frac{1}{2} < t < \frac{3}{4}$$

$$\int_0^{1/2}(4x^3+3x^2+2x+1)dx < \text{area} < \int_0^{3/4}(4x^3+3x^2+2x+1)dx$$

$$[x^4+x^3+x^2+x]_0^{1/2} < \text{area} < [x^4+x^3+x^2+x]_0^{3/4}$$

$$\frac{1}{16}+\frac{1}{8}+\frac{1}{4}+\frac{1}{2} < \text{area} < \frac{81}{256}+\frac{27}{64}+\frac{9}{16}+\frac{3}{4}$$

$$\frac{15}{16} < \text{area} < \frac{525}{256}$$

Hence, the correct answer is option (A).

8. The function $f'(x)$ is

(A) increasing in $\left(-t, -\frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4}, t\right)$

(B) decreasing in $\left(-t, -\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4}, t\right)$

(C) increasing in $(-t, t)$

(D) decreasing in $(-t, t)$

Solution:

$$f(x) = 4x^3 + 3x^2 + 2x + 1$$
$$f'(x) = 12x^2 + 6x + 2$$
$$f''(x) = 2[12x + 3] = 0 \Rightarrow x = -1/4$$
$$f'''(x) = 24$$

So, the funtion is decreasing in $(-t, t)$.

Hence, the correct answer is option (D).

9. Let the straight line $x = b$ divides the area enclosed by $y = (1-x)^2$, $y = 0$ and $x = 0$ into two parts $R_1(0 \le x \le b)$ and $R_2(b \le x \le 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals

(A) $\frac{3}{4}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{4}$

[IIT-JEE 2011]

Solution: See Fig. 24.42.

Therefore,

$$\int_0^b (1-x)^2 dx - \int_b^1 (1-x)^2 dx = \frac{1}{4}$$

$$\Rightarrow \left[\frac{(x-1)^3}{3}\right]_0^b - \left[\frac{(x-1)^3}{3}\right]_b^1 = \frac{1}{4}$$

$$\Rightarrow \frac{(b-1)^3}{3} + \frac{1}{3} - \left(0 - \frac{(b-1)^3}{3}\right) = \frac{1}{4}$$

$$\Rightarrow \frac{2(b-1)^3}{3} = -\frac{1}{12} \Rightarrow (b-1)^3 = -\frac{1}{8} \Rightarrow b = \frac{1}{2}$$

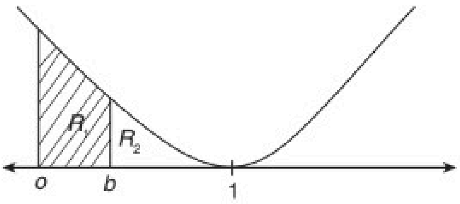

Figure 24.42

Hence, the correct answer is option (B).

10. Let f: $[-1, 2] \to [0, \infty]$ be a continuous function such that $f(x) = f(1-x)$ for all $x \in [-1, 2]$. Let $R_1 = \int_{-1}^{2} xf(x)dx$, and R_2 be the area of the region bounded by $y = f(x)$, $x = -1$, $x = 2$, and the x-axis. Then

(A) $R_1 = 2R_2$ **(B)** $R_1 = 3R_2$

(C) $2R_1 = R_2$ **(D)** $3R_1 = R_2$

[IIT-JEE 2011]

Solution:

$$R_1 = \int_{-1}^{2} xf(x)dx = \int_{-1}^{2}(2-1-x)f(2-1-x)dx$$

$$= \int_{-1}^{2}(1-x)f(1-x)dx = \int_{-1}^{2}(1-x)f(x)dx$$

Hence, $2R_1 = \int_{-1}^{2} f(x)dx = R_2$.

Hence, the correct answer is option (C).

11. Let S be the area of the region enclosed by $y = e^{-x^2}$, $y = 0$, $x = 0$ and $x = 1$. Then

(A) $S \ge \frac{1}{e}$ **(B)** $S \ge 1 - \frac{1}{e}$

(C) $S \le \frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$ **(D)** $S \le \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

[IIT-JEE 2012]

Solution: See Fig. 24.43.

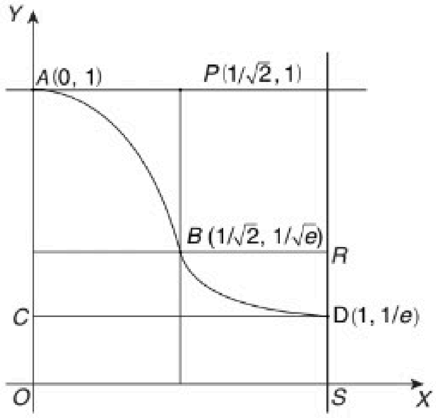

Figure 24.43

$$S > \frac{1}{e} \text{ (As area of rectangle } OCDS = 1/e)$$

Since,

$$e^{-x^2} \geq e^{-x} \ \forall \ x \in [0, 1]$$

$$\Rightarrow S > \int_0^1 e^{-x} dx = \left(1 - \frac{1}{e}\right)$$

Area of rectangle $OAPQ$ + Area of rectangle $QBRS > S$

$$S < \frac{1}{\sqrt{2}}(1) + \left(1 - \frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{e}}\right)$$

Since,

$$\frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right) < 1 - \frac{1}{e}$$

Hence, the correct answers are options (A), (B) and (D).

12. The area enclosed by the curves $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ over the interval $\left[0, \frac{\pi}{2}\right]$ is

(A) $4(\sqrt{2} - 1)$ **(B)** $2\sqrt{2}(\sqrt{2} - 1)$

(C) $2(\sqrt{2} + 1)$ **(D)** $2\sqrt{2}(\sqrt{2} + 1)$

[JEE ADVANCED 2013]

Solution: Figure 24.44 depicts the area enclosed by the given curves, we have

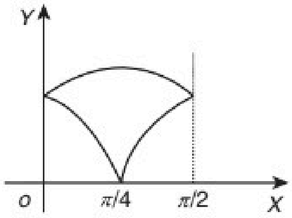

Figure 24.44

$$\int_0^{\pi/2} (\sin x + \cos x) dx - \left[\int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x) dx\right]$$

$$= -|\cos x|_0^{\pi/2} + |\sin x|_0^{\pi/2} - \left[|\sin x|_0^{\pi/4} + |\cos x|_0^{\pi/4} - |\cos x|_{\pi/4}^{\pi/2} - |\sin x|_{\pi/4}^{\pi/2}\right]$$

$$= -(0-1) + (1-0) - \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 - \left(0 - \frac{1}{\sqrt{2}}\right) - \left(1 - \frac{1}{\sqrt{2}}\right)\right]$$

$$= 2 - \left[\sqrt{2} - 1 + \frac{1}{\sqrt{2}} - 1 + \frac{1}{\sqrt{2}}\right] = 2 - \left[2\sqrt{2} - 2\right]$$

$$= 4 - 2\sqrt{2} = 2\sqrt{2}(\sqrt{2} - 1)$$

Hence, the correct answer is option (B).

13. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \leq d_1(P) + d_2(P) \leq 4$, is ______.

[JEE ADVANCED 2014]

Solution:

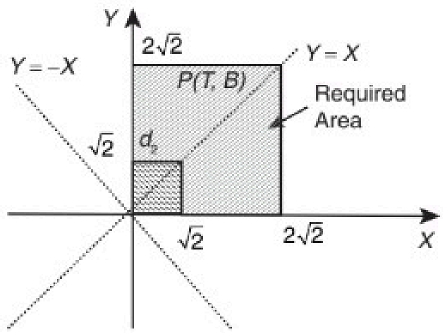

Figure 24.45

$$d_1 = \frac{|x - y|}{\sqrt{2}}$$

$$d_2 = \frac{|x + y|}{\sqrt{2}}$$

Therefore, according to the question (Fig. 24.45)

$$2 \leq \frac{|x - y|}{\sqrt{2}} + \frac{|x + y|}{\sqrt{2}} \leq 4$$

$$\Rightarrow 2\sqrt{2} \leq |x - y| + |x + y| \leq 4\sqrt{2} \quad (1)$$

Since $x, y \geq 0$ in the first quadrant.

When $x > y$ (or $y - x < 0$),

$$|x - y| = x - y \text{ and } |x + y| = x + y$$

Therefore, Eq. (1) is true given that,

$$2\sqrt{2} \leq x - y + x + y \leq 4\sqrt{2} \Rightarrow \sqrt{2} \leq x \leq 2\sqrt{2}$$

checking with (2, 1) in region $x > y$, i.e. $2 > 1$.

Therefore, we shade area below $y = x$ from $[\sqrt{2}, 2\sqrt{2}]$.

Area of this region $= \frac{1}{2}(2\sqrt{2} \times 2\sqrt{2}) - \frac{1}{2}\sqrt{2} \times \sqrt{2} = 4 - 1 = 3$ sq. units

By symmetry about $y = x$, total area required = 6 sq. units

Hence, the correct answer is (6).

14. Let $F(x)=\int_{x}^{x^2+\frac{\pi}{6}} 2\cos^2 t\,dt$ for all $x\in\mathbb{R}$ and $f:\left[0,\frac{1}{2}\right]\to[0,\infty)$ be a continuous function. For $a\in\left[0,\frac{1}{2}\right]$, if $F'(a)+2$ is the area of the region bounded by $x=0$, $y=0$, $y=f(x)$ and $x=a$, then $f(0)$ is ______.

[JEE ADVANCED 2015]

Solution: We have

$$F'(a)+2=\int_0^a f(x)dx$$

Differentiating both sides, we get

$$F''(a)=f(a)$$

Now,

$$F(x)=\int_{x}^{x^2+\pi/6} 2\cos^2 t dt$$

$$F'(x)=2x\cdot 2\cos^2\left(x^2+\frac{\pi}{6}\right)-2\cos^2 x$$

$$\Rightarrow F''(x)=-16x^2\cos\left(x^2+\frac{\pi}{6}\right)\sin\left(x^2+\frac{\pi}{6}\right)+4\cos x\sin x$$
$$+4\cos^2\left(x^2+\frac{\pi}{6}\right)$$

$$\Rightarrow F''(a)=-16a^2\cos\left(a^2+\frac{\pi}{6}\right)\sin\left(a^2+\frac{\pi}{6}\right)+4\cos a\sin a$$
$$+4\cos^2\left(a^2+\frac{\pi}{6}\right)$$

$$\Rightarrow f(0)=4\cos^2\left(\frac{\pi}{6}\right)=4\left(\frac{3}{4}\right)=3$$

Hence, the correct answer is (3).

15. Match the Column I to Column II.

	Column I	Column II
(A)	In ΔXYZ, let a, b and c be the lengths of the sides opposite to the angles X, Y and Z, respectively. If $2(a^2-b^2)=c^2$ and $\lambda=\frac{\sin(X-Y)}{\sin Z}$, then possible values of n for which $\cos(n\pi\lambda)=0$ is (are)	**(P)** 1
(B)	In ΔXYZ, let a, b and c be the lengths of the sides opposite to the angles X, Y and Z, respectively. If $1+\cos 2X-2\cos 2Y=2\sin X\sin Y$, then possible value(s) of $\frac{a}{b}$ is (are)	**(Q)** 2
(C)	In $\mathbb{R}^3$, let $\sqrt{3}\hat{i}+\hat{j}$, $\hat{i}+\sqrt{3}\hat{j}$, and $\beta\hat{i}+(1-\beta)\hat{j}$ be the position vectors of X, Y and Z with respect to the origin O, respectively. If the distance of Z from the bisector of the acute angle of $\overrightarrow{OX}$ with $\overrightarrow{OY}$ is $\frac{3}{\sqrt{2}}$, then possible value(s) of $\lvert\beta\rvert$ is (are)	**(R)** 3
(D)	Suppose that $F(\alpha)$ denotes the area of the region bounded by $x=0$, $x=2$, $y^2=4x$ and $y=\lvert\alpha x-1\rvert+\lvert\alpha x-2\rvert+\alpha x$, where $\alpha\in\{0,1\}$ Then the value(s) of $F(\alpha)+\frac{8}{3}\sqrt{2}$, when $\alpha=0$ and $\alpha=1$, is (are)	**(S)** 5
		(T) 6

[JEE ADVANCED 2015]

Solution: See Fig. 24.46.

$$2(a^2-b^2)=c^2 \quad (1)$$

$$\lambda=\frac{\sin(x-y)}{\sin z} \quad (2)$$

$$\cos(n\pi\lambda)=0 \quad (3)$$

$$\Rightarrow n\lambda=\frac{(2m+1)}{2} \quad (4)$$

From Eq. (2),

$$\lambda=\frac{\sin x\cos y-\cos x\sin y}{\sin z}$$

$$\Rightarrow \lambda=\frac{a\cos y-b\cos x}{c} \quad \text{(By Sine formula)}$$

$$\Rightarrow \lambda=\frac{a\left(\frac{a^2+c^2-b^2}{2ac}\right)-b\left(\frac{b^2+c^2-a^2}{2bc}\right)}{c}$$

$$\Rightarrow \lambda=\frac{2(a^2-b^2)}{2c^2}=\frac{1}{2} \quad (5)$$

Therefore, from Eqs. (4) and (5),

$$\frac{n}{2}=\frac{2m+1}{2}\Rightarrow n=(2m+1)$$

So,

$$\text{(A)}\rightarrow\text{(P), (R), (S)}$$

Checking option (B):

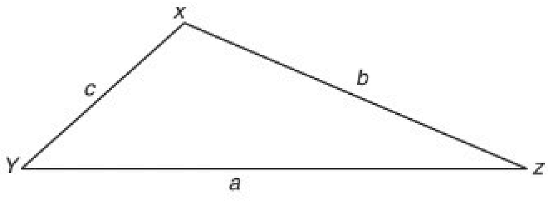

Figure 24.46

$$1+\cos 2x-2\cos 2y=2\sin x\sin y$$
$$\Rightarrow 2\cos^2 x-2(2\cos^2 y-1)=2\sin x\sin y$$
$$\Rightarrow 2\cos^2 x-4\cos^2 y+2=2\sin x\sin y$$
$$\Rightarrow 2\sin^2 y-2\sin x\sin y+\sin x\sin y-\sin^2 x=0$$
$$\Rightarrow 2\sin y(\sin y-\sin x)+\sin x(\sin y-\sin x)=0$$
$$\Rightarrow (\sin y-\sin x)(2\sin y+\sin x)=0$$
$$\Rightarrow b=a \text{ or } 2b=-a \text{ (impossible)}$$
$$\Rightarrow \frac{a}{b}=1$$

So,

$$\text{(B)}\rightarrow\text{(P)}$$

Checking option (C): See Fig. 24.47.

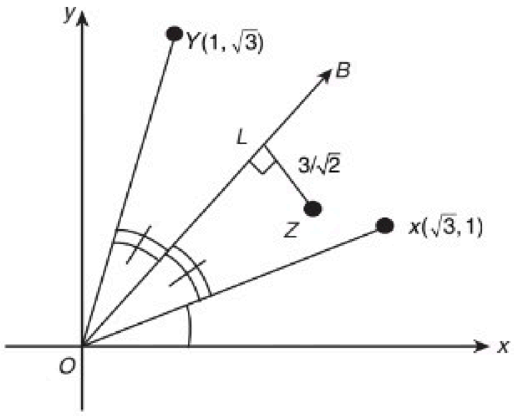

Figure 24.47

Vector along the bisector of acute angle between $\overline{OX}$ and $\overline{OY}$ is,

$$\frac{\sqrt{3}\hat{i}+\hat{j}}{2}+\frac{\hat{i}+\sqrt{3}\hat{j}}{2}=\frac{(\sqrt{3}+1)}{2}(\hat{i}+\hat{j})$$

Slope of $\overline{OB}=\tan(\pi/4)=1$

$\Rightarrow$ Equation of OB is $y=x$

Since,

$$ZL=3/\sqrt{2}, \Rightarrow \frac{|\beta-(1-\beta)|}{\sqrt{2}}=\frac{3}{\sqrt{2}}$$

$\Rightarrow |2\beta-1|=3$
$\Rightarrow (2\beta-1)=\pm 3$
$\Rightarrow \beta=2$ or $\beta=-1$
$\Rightarrow |\beta|=1$ or 2

Therefore, (C)$\rightarrow$ (P), (Q).

Checking option (D): See Fig. 24.48.

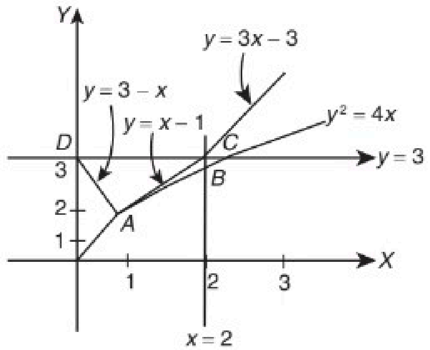

Figure 24.48

$$y=|\alpha x-1|+|\alpha x-2|+\alpha x;\ \alpha\in\{0, 1\}$$

Case (I) For $\alpha=0, y=3$

Case (II) For $\alpha=1, y=|x-1|+|x-2|+x$

$$\Rightarrow y=\begin{cases}3-x; & x\le 1\\ x+1; & 1<x<2\\ 3x-3; & x\ge 2\end{cases}$$

Therefore,

$$F(0)=\int_0^2(3-2\sqrt{x})dx=\left[3x-\frac{4}{3}x^{3/2}\right]_0^2$$

$$=\left[6-\frac{4}{3}(2\sqrt{2})\right]=6-\frac{8}{3}\sqrt{2}$$

$$\Rightarrow F(0)+\frac{8}{3}\sqrt{2}=6\Rightarrow (T)$$

and $F(1)=F(0)-$ area of ΔACD

$$=\left(6-\frac{8}{3}\sqrt{2}\right)-\frac{1}{2}(2)(1)=5-\frac{8}{3}\sqrt{2}$$

$$\Rightarrow F(1)+\frac{8}{3}\sqrt{2}=5\Rightarrow\ (S)$$

Therefore,

(D) $\rightarrow$ (T), (S)

Hence, the correct matches are (A) $\rightarrow$ (P), (R), (S); (B) $\rightarrow$ (P); (C) $\rightarrow$ (P), (Q); (D) $\rightarrow$ (S), (T).

16. The area of the region $\{(x,y)\in\mathbb{R}^2: y\ge\sqrt{|x+3|}, 5y\le x+9\le 15\}$ is equal to

(A) $\frac{1}{6}$ **(B)** $\frac{4}{3}$

(C) $\frac{3}{2}$ **(D)** $\frac{5}{3}$

[JEE ADVANCED 2016]

Solution: It is given that

$$y\ge\sqrt{|x+3|}$$

That is,

$$\sqrt{|x+3|}=\begin{cases}\sqrt{(x+3)}, & x\ge -3\\ \sqrt{-x-3}, & x<-3\end{cases}$$

It is also given that

$$5y\le x+9\le 15$$

That is,

$$x+9\le 15\Rightarrow x\le 6$$
$$5y\le 15\Rightarrow y\le 3$$
$$5y\le x+9$$

From Fig. 24.49, we have

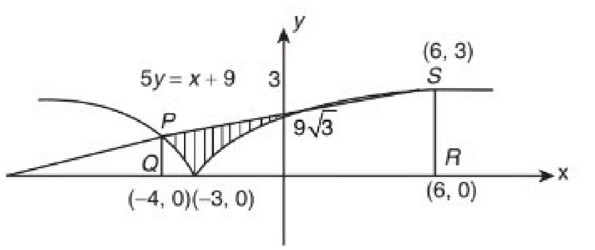

Figure 24.49

$$5y = x + 9 \tag{1}$$

and

$$y = \sqrt{-x-3}$$

That is,

$$\left(\frac{x+9}{5}\right)^2 = -(x+3)$$

$$x^2 + 81 + 18x = -25x - 75$$

$$x^2 + 43x + 156 = 0$$

$$(x+39)(x+4) = 0 \Rightarrow x = -4$$

(Since, $x \neq -39$)

Substituting the value of x in Eq. (1), we get the coordinates of point P as follows:

$$5y = -4 + 9 \Rightarrow y = 1 \Rightarrow P(-4, 1)$$

The area of trapezium $PQRS$ is

$$\frac{1}{2} \times 10 \times 4 = 20$$

Hence, the area of the given region is

$$20 - \int_{-4}^{-3} \sqrt{-x-3}\,dx - \int_{-3}^{6} \sqrt{x+3}\,dx$$

$$= 20 + \frac{2}{3}(-x-3)^{3/2}\Big|_{-4}^{-3} - \frac{2}{3}(x+3)^{3/2}\Big|_{-3}^{6}$$

$$= 20 + \frac{2}{3}(0-1) - \frac{2}{3}9^{3/2}$$ 1

$$= 20 - \frac{2}{3} - \frac{2}{3} \times 27$$

$$= 2 - \frac{2}{3} = \frac{4}{3} \text{ sq. units.}$$ 1

Hence, the correct answer is option (B).

1