6. Let y be the function that passes through (1, 2) having slope $(2x + 1)$. The area bounded between the curve and x-axis is

(A) 6 sq. units **(B)** 5/6 sq. units

(C) 1/6 sq. units **(D)** None of these **Ans.** (C)

7. Area bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is given by

(A) $\frac{8}{9}$ sq. units **(B)** $\frac{9}{8}$ sq. units

(C) $\frac{4}{3}$ sq. units **(D)** None of these **Ans.** (B)

8. The area of the region bounded by the curve $y = x|x|$, x-axis and the ordinates $x = 1, x = -1$ is given by

(A) Zero **(B)** $\frac{1}{3}$

(C) $\frac{2}{3}$ **(D)** 1 **Ans.** (C)

9. If the area bounded by $y = ax^2$ and $x = ay^2$, $a > 0$, is 1, then $a =$

(A) 1 **(B)** $\frac{1}{\sqrt{3}}$

(C) $\frac{1}{3}$ **(D)** None of these **Ans.** (B)

10. The area bounded by the curves $y = \sqrt{x}$, $2y + 3 = x$ and x-axis in the first quadrant is

(A) 9 **(B)** $\frac{27}{4}$ **(C)** 36 **(D)** 18 **Ans.** (A)

11. The area enclosed between the curve $y = \log_e(x + e)$ and the coordinate axes is

(A) 3 **(B)** 4 **(C)** 1 **(D)** 2 **Ans.** (C)

12. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4, y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom, then $S_1 : S_2 : S_3$ is

(A) 2 : 1 : 2 **(B)** 1 : 1 : 1 **(C)** 1 : 2 : 1 **(D)** 1 : 2 : 3 **Ans.** (B)

Additional Solved Examples

1. The total area enclosed by the lines $y = |x|$, $|x| = 1$ and $y = 0$ is

(A) 1 **(B)** 2 **(C)** $\frac{1}{2}$ **(D)** None of these

Solution: See Fig. 24.17.

$$y = |x|, |x| = 1, y = 0$$

$$\text{Total area } = \frac{1}{2}\times 1\times 1 + \frac{1}{2}\times 1\times 1 = \frac{1}{2} + \frac{1}{2} = 1 \text{ sq. units}$$

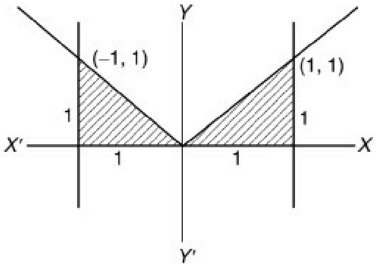

Figure 24.17

Hence, the correct answer is option (A).

2. The area bounded by $y = \frac{|x|}{x}$, $x \neq 0$, and the lines $y(x - 1)(x - 3) = 0$ is

(A) 3 **(B)** 1 **(C)** 2 **(D)** None of these

Solution: See Fig. 24.18.

$$y = \frac{|x|}{x}, x \neq 0,$$
$$y(x-1)(x-3) = 0$$
$$\text{Area} = 2 \times 1$$
$$= 2 \text{ sq. units}$$

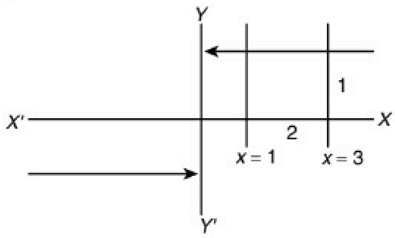

Figure 24.18

Hence, the correct answer is option (C).

3. The area bounded by the curve $|x| = \cos^{-1}y$ and the line $|x| = 1$ and the x-axis is

(A) cos 1 **(B)** sin 1 **(C)** 2 cos 1 **(D)** 2 sin 1

Solution: See Fig. 24.19.

$|x| = \cos^{-1}y$ and line $|x| = 1$ and x-axis

$y = \cos|x|$

$|x| = 1$, x-axis

$$\text{Area} = 2\int_0^1 \cos x\, dx$$

$$= 2[\sin x]_0^1 = 2\sin 1 \text{ sq. units}$$

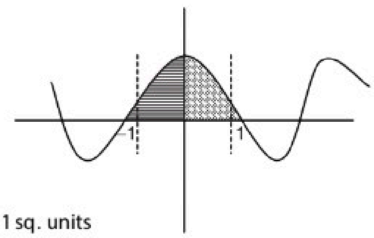

Figure 24.19

Hence, the correct answer is option (D).

4. Area bounded by the curve $f(x) = \begin{cases} \log_e|x|, & |x| \geq \frac{1}{e} \\ |x| - 1 - \frac{1}{e}, & |x| < \frac{1}{e} \end{cases}$ and x-axis is

(A) $\frac{1}{e^2} + 2 - \frac{2}{e}$ **(B)** $\frac{1}{e^2} + 2 + \frac{2}{e}$

(C) $\frac{1}{e^2} + \frac{2}{e}$ **(D)** None of these

Solution: See Fig. 24.20.

$$f(x) = \begin{cases} \log_e|x|, & |x| \geq \frac{1}{e} \\ |x| - 1 - \frac{1}{e}, & |x| < \frac{1}{e} \end{cases}$$

$$\text{Area required} = 2\left[\int_0^{1/e} -\left(x - 1 - \frac{1}{e}\right)dx + \int_{1/e}^{1} -\ln x\, dx\right]$$

$$= 2\left[-\left\{\frac{x^2}{2} - \left(1 + \frac{1}{e}\right)x\right\}_0^{1/e} - (x(\ln x - 1))_{1/e}^{1}\right]$$

$$= -2\left[\frac{1}{2e^2} - \frac{1}{e} - \frac{1}{e^2}\right] - 2\left[(-1) + \left(\frac{2}{e}\right)\right]$$

$$= -\frac{1}{e^2} + \frac{2}{e} + \frac{2}{e^2} + 2 - \frac{4}{e}$$

$$= \frac{1}{e^2} + 2 - \frac{2}{e} \text{ sq. units}$$

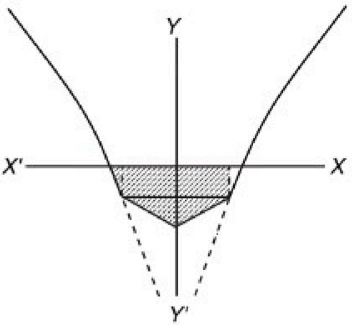

Figure 24.20

Hence, the correct answer is option (A).

5. The whole area of the curves $x = a\cos^3 t$, $y = b\sin^3 t$ is given by

(A) $\frac{3}{8}\pi ab$ **(B)** $\frac{5}{8}\pi ab$ **(C)** $\frac{1}{8}\pi ab$ **(D)** None of these

Solution:

$$\text{Area} = 4\int_0^a y\frac{dx}{dt}\cdot dt$$

$$= 4\int_{\pi/2}^{0} -3ab\sin^4 t\cdot\cos^2 t\cdot dt$$

$$= 4\times 3ab\cdot\frac{3}{6}\cdot\frac{1}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}$$

$$= \frac{3}{8}\pi\, ab \text{ sq. units}$$

Hence, the correct answer is option (A).

6. Area common to the curves $y^2 = ax$ and $x^2 + y^2 = 4ax$ is equal to

(A) $(9\sqrt{3} + 4\pi)\frac{a^2}{3}$ **(B)** $(9\sqrt{3} + 4\pi)a^2$

(C) $(9\sqrt{3} - 4\pi)\frac{a^2}{3}$ **(D)** None of these

Solution: See Fig. 24.21.

$y^2 = ax, x^2 + y^2 - 4ax = 0$

$y^2 = 4ax - x^2$

$x^2 + ax - 4ax = 0$

$x^2 - 3ax = 0$

$x(x - 3a) = 0$

$x = 0, x = 3a$

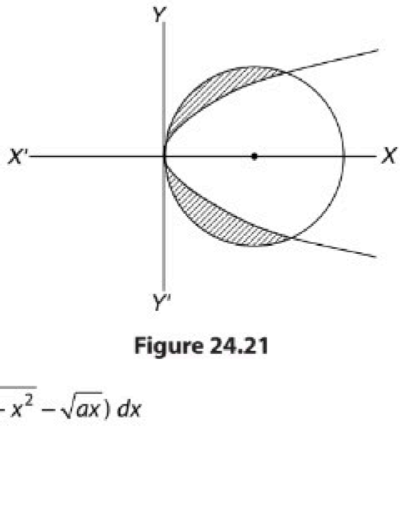

Figure 24.21

$$\text{Required area} = 2\int_0^{3a}(\sqrt{4ax - x^2} - \sqrt{ax})\,dx$$

$$= 2\int_0^{3a}(\sqrt{4a^2 - (x-2a)^2} - \sqrt{ax})\,dx = \frac{(8\pi - 9\sqrt{3})a^2}{3} \text{ sq. units}$$

Hence, the correct answer is option (D).

7. The area $\{(x, y); x^2 \le y \le \sqrt{x}\}$ is equal to

(A) $\frac{1}{3}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{1}{6}$ **(D)** None of these

Solution: See Fig. 24.22.

$$\{(x, y); x^2 \le y \le \sqrt{x}\}$$

$$\text{Area} = \int_0^1(\sqrt{x} - x^2)dx$$

$$= \left[\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right]_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3} \text{ sq. units}$$

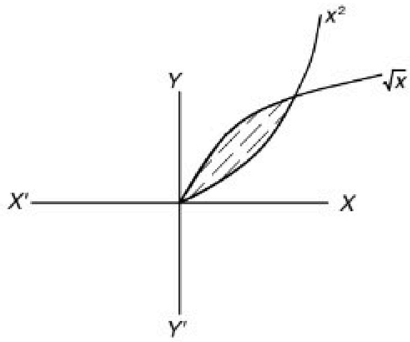

Figure 24.22

Hence, the correct answer is option (A).

8. The area enclosed by the curve $y = x^5$, the x-axis and the ordinates $x = -1, x = 1$ is

(A) $\frac{1}{2}$ **(B)** 1 **(C)** $\frac{1}{3}$ **(D)** 0

Solution: See Fig. 24.23.

$$y = x^5, x = \pm 1$$

$$\text{Area} = 2\int_0^1 x^5 dx = 2\left.\frac{x^6}{6}\right|_0^1 = \frac{2}{6} = \frac{1}{3} \text{ sq. units}$$

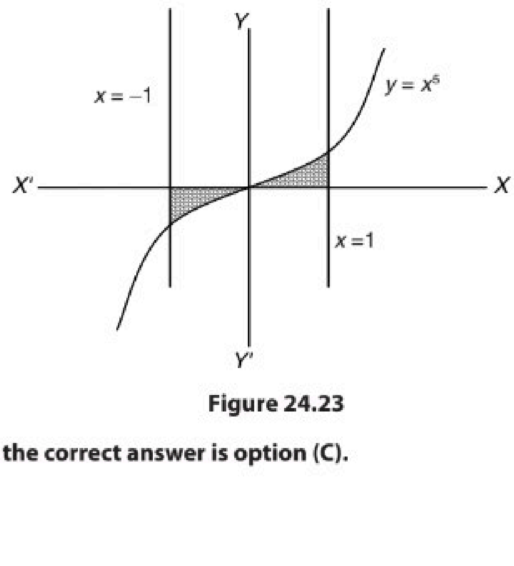

Figure 24.23

Hence, the correct answer is option (C).

9. The area bounded by the curve $y^2 = 9x$ and the lines $x = 1, x = 4$ and $y = 0$ in the first quadrant is

(A) 14 **(B)** 7 **(C)** 28 **(D)** None of these

Solution: See Fig. 24.24.

$$\text{Area} = \int_1^4 3\sqrt{x}\, dx = 2 \times 3 \frac{x^{3/2}}{3}\Bigg|_1^4 = 2\left[x^{3/2}\right]_1^4$$

$$= 2\left[2^3 - 1\right] = 14 \text{ sq. units}$$

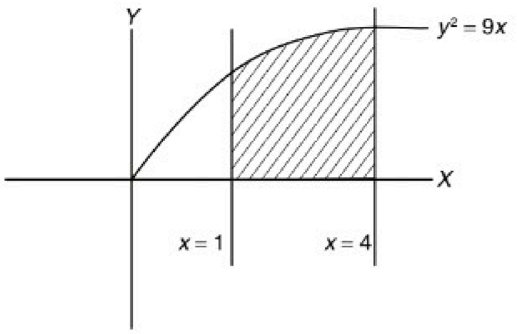

Figure 24.24

Hence, the correct answer is option (A).

10. The slope of the tangent to a curve $y = f(x)$ at $(x, f(x))$ is $2x + 1$. If the curve passes through the point (1, 2), then the area of the region bounded by the curve, the x-axis and the line $x = 1$ is

(A) $\frac{5}{6}$ **(B)** $\frac{6}{5}$ **(C)** $\frac{1}{6}$ **(D)** 6

Solution: See Fig. 24.25.

$$f'(x) = 2x + 1$$

$$\Rightarrow f(x) = x^2 + x + c$$

The curve passes through (1, 2), so

$$2 = 1 + 1 + c$$

$$\Rightarrow c = 0$$

$$f(x) = x^2 + x$$

$$\int_0^1 (x^2 + x)dx = \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^1 = \frac{1}{3} + \frac{1}{2} = \frac{5}{6} \text{ sq. units}$$

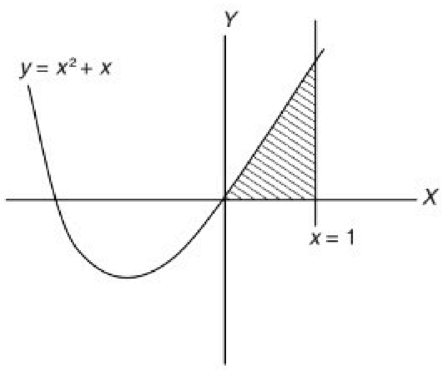

Figure 24.25

Hence, the correct answer is option (A).

11. The area bounded by the axes of reference and normal to $y = \log_e x$ at the point (1, 0) is

(A) 1 sq. units **(B)** 2 sq. units

(C) $\frac{1}{2}$ sq. units **(D)** None of these

Solution: See Fig. 24.26.

$$y = \ln x$$

$$\frac{dy}{dx} = \frac{1}{x}$$

At $x = 1$,

$$\text{slope of normal} = -1$$

$$y = -1(x - 1)$$

$$y + x = 1$$

$$\text{Area} = \frac{1}{2} \times 1 \times 1 = \frac{1}{2} \text{ sq. units}$$

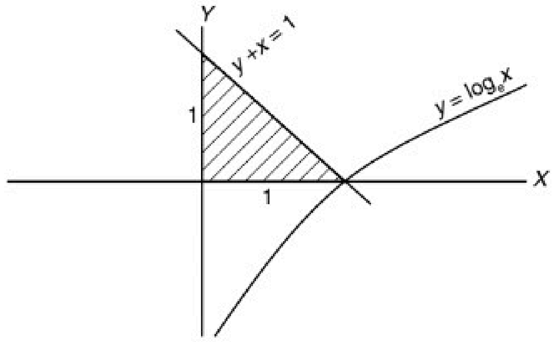

Figure 24.26

Hence, the correct answer is option (C).

12. The area bounded by the line $|x| + |y| = 1$ is

(A) 4 **(B)** 2 **(C)** 1 **(D)** None of these

Solution: See Fig. 24.27.

$$|x| + |y| = 1$$

$$\text{Area} = 4 \times \left(\frac{1}{2} \times 1 \times 1\right) = 2 \text{ sq. units}$$

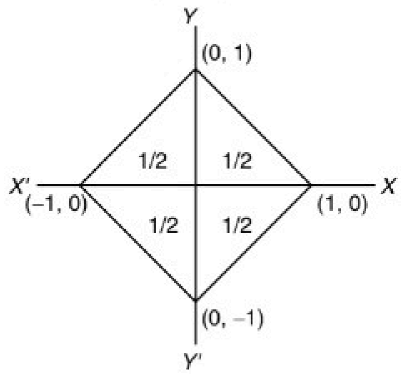

Figure 24.27

Hence, the correct answer is option (B).

13. If area bounded by curve $f(x)$ and x-axis, $x = 1$ to $x = b$ is $(b - 1)\sin(3b + 4)$, then $f(x)$ is

(A) $3x\cos(3x+4)+\sin(3x+4)$
(B) $3(x-1)\cos(3x+4)+\sin(3x+4)$
(C) $x\cos(3x+4)+\sin(3x+4)$
(D) None of these

Solution:

$$\int_1^b f(x)\,dx=(b-1)\sin(3b+4)$$

$$\int_1^x f(x)\,dx=(x-1)\sin(3x+4) \text{ (replacing } b \text{ by } x)$$

$$f(x)=3(x-1)\cos(3x+4)+\sin(3x+4)$$

Hence, the correct answer is option (B).

14. The area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is

(A) $\frac{15}{4}$ **(B)** $\frac{17}{4}$ **(C)** $\frac{9}{4}$ **(D)** None of these

Solution: See Fig. 24.28.

$$\text{Area } = \left|\int_{-2}^{0} x^3 dx\right| + \int_0^1 x^3 dx = \left|\frac{x^4}{4}\right|_{-2}^{0} + \left.\frac{x^4}{4}\right|_0^1$$

$$= \frac{16}{4}+\frac{1}{4}=\frac{17}{4} \text{ sq. units}$$

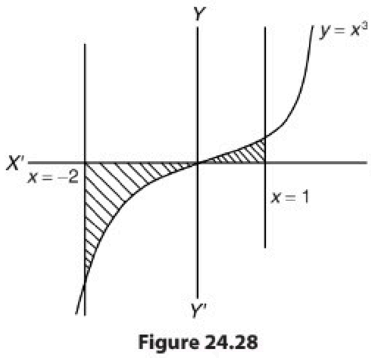

Figure 24.28

Hence, the correct answer is option (B).

15. The area of the region bounded by $y = |x - 1|$ and $y = 1$ is

(A) $\frac{1}{2}$ **(B)** 1
(C) 2 **(D)** None of these

Solution:

$$\text{Area} = \frac{1}{2}\times 1\times 2$$

Solution: Solving $y^2 = x$ and $y = x$, we get, $y = 0, x = 0, y = 1, x = 1$
Therefore,

$$\text{Area} = \int_0^1 (\sqrt{x}-x)dx = \left[\frac{2}{3}x^{3/2}-\frac{x^2}{2}\right]_0^1 = \frac{2}{3}-\frac{1}{2}=\frac{1}{6} \text{ sq. units}$$

Hence, the correct answer is option (C).

2. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to

(A) $\frac{5}{3}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{4}{3}$

[AIEEE 2008]

Solution: Solving the equations, we get the points of intersection as (−2, 1) and (−2, −1). The bounded region is shown as shaded region in Fig. 24.29.

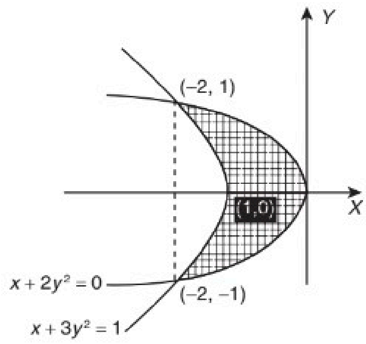

Figure 24.29

The required area is

$$2\int_0^1\left[(1-3y^2)-(-2y^2)\right]dy = 2\int_0^1 (1-y^2)dy = 2\left[y-\frac{y^3}{3}\right]_0^1$$

$$= 2\times\frac{2}{3}=\frac{4}{3} \text{ sq. units}$$

Hence, the correct answer is option (D).

3. The area of the region bounded by the parabola $(y - 2)^2 = x - 1$, the tangent to the parabola at the point (2, 3) and the x-axis is

(A) 3 **(B)** 6 **(C)** 9 **(D)** 12

[AIEEE 2009]

Solution: Equation of tangent at (2, 3) for the parabola, $(y-2)^2 = x - 1$, is $S_1 = 0$, which implies that $x - 2y + 4 = 0$.

Alternate solution:

The area is

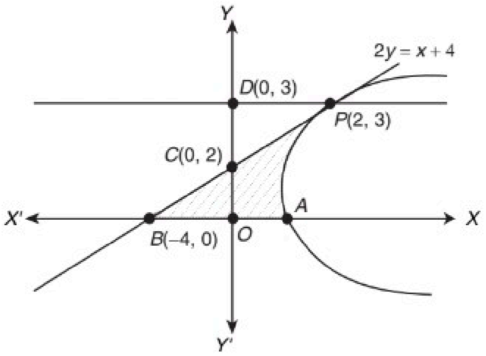

Figure 24.30

$$A\int_0^3 (2y-4-y^2+4y-5)dy = \int_0^3 (-y^2+6y-9)dy$$

$$= -\int_0^3 (3-y)^2 dy = \left[\frac{(y-3)^3}{3}\right]_0^3 = \frac{27}{3} = 9 \text{ sq. units}$$

Hence, the correct answer is option (C).

4. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is

(A) $4\sqrt{2}+2$ **(B)** $4\sqrt{2}-1$

(C) $4\sqrt{2}+1$ **(D)** $4\sqrt{2}-2$

[AIEEE 2010]

Solution: The required area is

$$\int_0^{\frac{\pi}{4}} (\cos x - \sin x)dx + \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)dx + \int_{\frac{5\pi}{4}}^{\frac{3\pi}{2}} (\cos x - \sin x)dx$$

$$= [\sin x + \cos x]_0^{\frac{\pi}{4}} + [-\cos x - \sin x]_{\frac{\pi}{4}}^{\frac{5\pi}{4}} + [\sin x + \cos x]_{\frac{5\pi}{4}}^{\frac{3\pi}{2}}$$

$$= (\sqrt{2}-1) + 2\sqrt{2} + (-1+\sqrt{2}) = 4\sqrt{2}-2 \text{ sq. units}$$

Hence, the correct answer is option (D).

5. The area of the region enclosed by the curves $y = x, x = e, y = \frac{1}{x}$ and the positive x-axis is

(A) 1 sq. units **(B)** $\frac{3}{2}$ sq. units

(C) $\frac{5}{2}$ sq. units **(D)** $\frac{1}{2}$ sq. units

[AIEEE 2011]

Solution: From Figure 24.31, we have,

$$\text{Area} = \int_0^1 x\,dx + \int_1^e \frac{1}{x}dx = \frac{1}{2} + 1 = \frac{3}{2} \text{ sq. units}$$

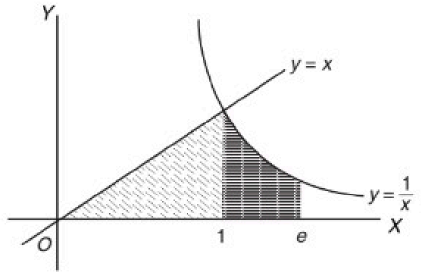

Figure 24.31

Hence, the correct answer is option (B).

6. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$, and the straight line $y = 2$ is

(A) $20\sqrt{2}$ **(B)** $\frac{10\sqrt{2}}{3}$

(C) $\frac{20\sqrt{2}}{3}$ **(D)** $10\sqrt{2}$

[AIEEE 2012]

Solution: From Figure 24.32, the required area is calculated as

$$A = 2\left[\int_0^2 \left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right)dy\right] = 2\int_0^2 \frac{5\sqrt{y}}{2}dy = 5\left[\frac{y^{3/2}}{3/2}\right]_0^2$$

$$= \frac{10}{3}[2^{3/2} - 0] = \frac{20\sqrt{2}}{3} \text{ sq. units}$$

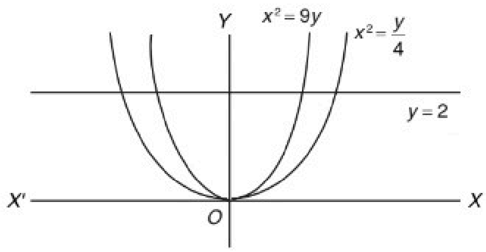

Figure 24.32

Hence, the correct answer is option (C).

7. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant is

(A) 36 **(B)** 18

(C) $\frac{27}{4}$ **(D)** 9

[JEE MAIN 2013]

Solution: First solving the equations, we have

$$2\sqrt{x} = x - 3 \quad (1)$$

Squaring on both sides of Eq. (1), we get

$$4x = x^2 - 6x + 9 \Rightarrow x^2 - 10x + 9 \Rightarrow x = 9, \; x = 1$$

Since $x = 1$ intersects the parabola below the x-axis, this point is extraneous.

So, for $x = 9$ we have, $y = 3$.

Therefore, the required area under the curve (see Fig. 24.33) is

$$\int_0^3 \left[(2y+3) - y^2\right] dy \Rightarrow \left[y^2 + 3y - \frac{y^3}{3}\right]_0^3 = 9 + 9 - 9 = 9 \text{ sq. units}$$

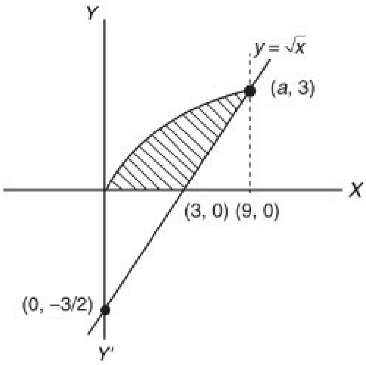

Figure 24.33

Hence, the correct answer is option (D).

8. The area of the region described by $A = \{(x, y): x^2 + y^2 \le 1$ and $y^2 \le 1 - x\}$ is

(A) $\frac{\pi}{2} - \frac{2}{3}$ **(B)** $\frac{\pi}{2} + \frac{2}{3}$

(C) $\frac{\pi}{2} + \frac{4}{3}$ **(D)** $\frac{\pi}{2} - \frac{4}{3}$

[JEE MAIN 2014 (OFFLINE)]

Solution: See Fig. 24.34.

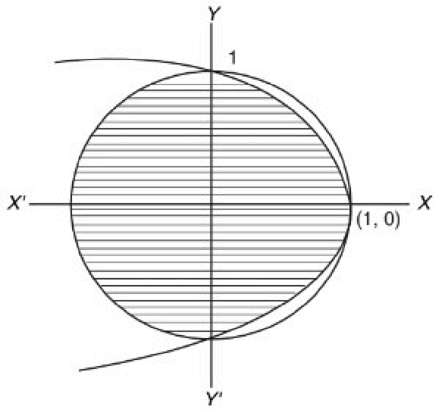

Figure 24.34

$$y^2 = 1 - x \Rightarrow x = 1 - y^2$$

Required area $= \frac{1}{2}(\pi \times 1^2) + 2\int_0^1 (1 - y^2) dy$

$$= \frac{\pi}{2} + 2\left[y - \frac{y^3}{3}\right]_0^1 = \frac{\pi}{2} + 2\left[\left(1 - \frac{1}{3}\right) - 0\right] = \frac{\pi}{2} + \frac{4}{3} \text{ sq. units}$$

Hence, the correct answer is option (C).

9. Let $A = \{(x, y): y^2 \le 4x,\ y - 2x \ge -4\}$. Then the area (in square units) of the region A is

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11

[JEE MAIN 2014 (ONLINE SET-1)]

Solution: See Fig. 24.35. Finding points of intersection,

$$\frac{y^2}{4} = \frac{y+4}{2}$$

Therefore,

$$2y^2 = 4y + 16 \text{ or } y^2 = 2y + 8 \Rightarrow y^2 - 2y - 8 = 0$$

$$y = \frac{2 \pm \sqrt{4+32}}{2} = \frac{2 \pm 6}{2} = 4, -2$$

Therefore,

$$x = 4, 1 \text{ and } P \text{ is } (1, -2) \text{ and } Q \text{ is } (4, 4)$$

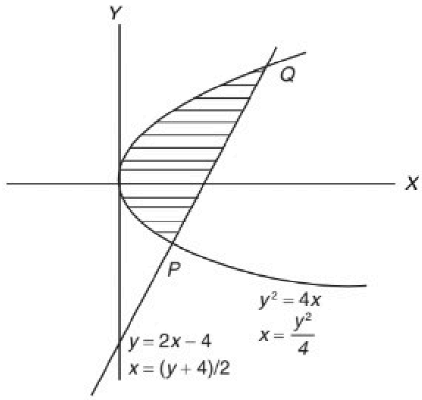

Figure 24.35

$$\text{Required area} = \int_{-2}^{4} \left\{\left(\frac{4+y}{2}\right) - \frac{y^2}{4}\right\} dy$$

$$= \int_{-2}^{4} \left(2 + \frac{1}{2}y - \frac{y^2}{4}\right) dy = \left[2y + \frac{y^2}{4} - \frac{y^3}{12}\right]_{-2}^{4}$$

$$= \left(8 + 4 - \frac{64}{12}\right) - \left(2(-2) + \frac{4}{4} + \frac{8}{12}\right)$$

$$= 12 + 4 - 1 - \left(\frac{72}{12}\right) = 15 - 6 = 9 \text{ sq. units}$$

Hence, the correct answer is option (B).

10. The area of the region above the x-axis bounded by the curve $y = \tan x, 0 \le x \le \frac{\pi}{2}$ and the tangent to the curve at $x = \frac{\pi}{4}$ is

(A) $\frac{1}{2}\left(\log 2 - \frac{1}{2}\right)$ **(B)** $\frac{1}{2}\left(\log 2 + \frac{1}{2}\right)$

(C) $\frac{1}{2}(1 - \log 2)$ **(D)** $\frac{1}{2}(1 + \log 2)$

[JEE MAIN 2014 (ONLINE SET-4)]

Solution: See Fig. 24.36.

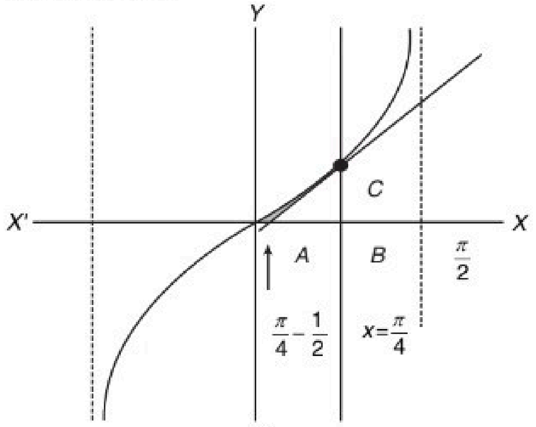

Figure 24.36

Required area $= \int_0^{\pi} \tan x$ − area under tangent at $\left(\frac{\pi}{4}, 1\right)$ (1)

Now slope of tangent is $\frac{d}{dx}\tan x$ at $x = \frac{\pi}{4} = \sec^2 x\Big|_{\text{at } x=\frac{\pi}{4}} = 2$

Therefore, equation of tangent is $y - 1 = 2\left(x - \frac{\pi}{4}\right)$ or $y = 2x + \left(1 - \frac{\pi}{2}\right)$

This tangent cuts x-axis when $y = 0$

Therefore,

$$x = \frac{\frac{\pi}{2} - 1}{2} = \frac{\pi}{4} - \frac{1}{2}$$

Thus, required area is

$$[\log \sec x]_0^{\frac{\pi}{4}} - \text{area triangle } ABC$$

$$= \log\sqrt{2} - 0 - \frac{1}{2}\times\left(\frac{\pi}{4} - \frac{\pi}{4} + \frac{1}{2}\right)\times 1 = \frac{1}{2}\left[\log 2 - \frac{1}{2}\right] \text{ sq. units}$$

Hence, the correct answer is option (A).

11. The area (in sq. units) of the region described by $\{(x, y): y^2 \le 2x$ and $y \ge 4x - 1\}$ is

(A) $\frac{5}{64}$ **(B)** $\frac{15}{64}$ **(C)** $\frac{9}{32}$ **(D)** $\frac{7}{32}$

[JEE MAIN 2015 (OFFLINE)]

Solution: See Fig. 24.37.

$$R = \{(x, y): y^2 \le 2x \text{ and } y \ge 4x - 1\}$$

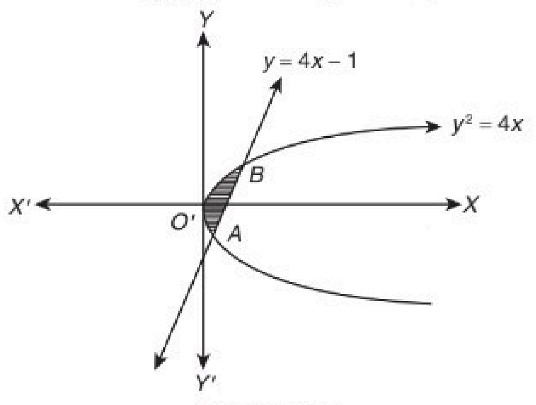

Figure 24.37

Finding points of intersection,

$$y^2 = 2\left(\frac{y+1}{4}\right) \Rightarrow 2y^2 = y + 1 \Rightarrow 2y^2 - y - 1 = 0$$

$$\Rightarrow (y-1)(2y+1) = 0$$

$$\Rightarrow y = 1 \text{ and } y = \frac{-1}{2}$$

$$\Rightarrow x = \frac{1}{2} \text{ and } x = \frac{1}{8}$$

So, point A is (1/8, − 1/2) and B is (1/2, 1).

$$R = \text{shaded area } \int_{y_A}^{y_B} (x_{\text{line}})dy - \int_{y_A}^{y_B} (x_{\text{parabola}})dy$$

$$= \int_{-1/2}^{1} \frac{1}{4}(y+1)dy - \int_{-1/2}^{1} \frac{y^2}{2}dy$$

$$= \frac{1}{4}\left[\frac{y^2}{2} + y\right]_{-1/2}^{1} - \frac{1}{2}\left[\frac{y^3}{3}\right]_{-1/2}^{1} = \frac{9}{32} \text{ sq. units}$$

Hence, the correct answer is option (C).

12. The area (in square units) of the region bounded by the curves $y + 2x^2 = 0$ and $y + 3x^2 = 1$ is equal to

(A) $\frac{3}{5}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{4}{3}$

[JEE MAIN 2015 (ONLINE SET-1)]

Solution: See Fig. 24.38.

$$C_1: y + 2x^2 = 0;$$

$$C_2: y + 3x^2 = 1$$

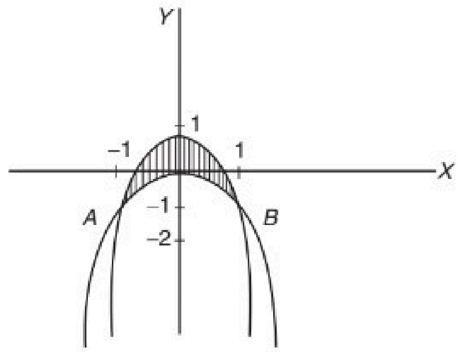

Figure 24.38

At the point of intersection of C_1 and C_2

$$-2x^2 = 1 - 3x^2 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

Therefore, A (−1, −2) and B (1, −2) are points of intersection as shown above.

So, required area is

$$2\int_{-1}^{0} [(1 - 3x^2) - (-2x^2)]dx$$

$$= 2\int_{-1}^{0} (1 - x^2)dx = 2\left[x - \frac{x^3}{3}\right]_{-1}^{0} = \frac{4}{3} \text{ sq. units}$$

Hence, the correct answer is option (D).