JEE MAIN

Single Correct Answer Type

1. $\int_0^{\pi}[\cot x]\,dx$, where [ ] denotes the greatest integer function, is equal to

(1) $\frac{\pi}{2}$ (2) 1 (3) -1 (4) $-\frac{\pi}{2}$ **(AIEEE 2009)**

2. Let $p(x)$ be a function defined on R such that $p'(x) = p'(1-x)$, for all $x \in [0, 1]$, $p(0) = 1$, and $p(1) = 41$. Then $\int_0^1 p(x)\,dx$ is equal to

(1) 42 (2) $\sqrt{41}$ (3) 21 (4) 41 **(AIEEE 2010)**

3. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}dx$ is

(1) $\log 2$ (2) $\pi \log 2$ (3) $\frac{\pi}{8}\log 2$ (4) $\frac{\pi}{2}\log 2$ **(AIEEE 2011)**

4. For $x \in \left(0, \frac{5\pi}{2}\right)$, define $f(x) = \int_0^x \sqrt{t}\sin t\,dt$. Then f has

(1) local maximum at π and local minima at 2π
(2) local maximum at π and 2π
(3) local minimum at π and 2π
(4) local minimum at π and local maximum at 2π

(AIEEE 2011)

5. If $g(x) = \int_0^x \cos^4 t\,dt$, then $g(x+\pi)$ equals

(1) $\frac{g(x)}{g(\pi)}$ (2) $g(x) + g(\pi)$ (3) $g(x) - g(\pi)$ (4) $g(x)\cdot g(\pi)$ **(AIEEE 2012)**

6. **Statement I:**

The value of the integral $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$.

Statement II:

$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

(1) Statement I is true; statement II is true; statement II is a correct explanation for statement I
(2) Statement I is true; statement II is true; statement II is a not a correct explanation for statement I.
(3) Statement I is true; statement II is false.
(4) Statement I is false; statement II is true.

(JEE Main 2013)

7. The intercepts on x-axis made by tangents to the curve, $y = \int_0^x |t|\,dt$, $x \in R$, which are parallel to the line $y = 2x$, are equal to

(1) ± 1 (2) ± 2 (3) ± 3 (4) ± 4

(JEE Main 2013)

8. The integral $\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$ equals

(1) $\pi - 4$ (2) $\frac{2\pi}{3} - 4 - 4\sqrt{3}$ (3) $4\sqrt{3} - 4$ (4) $4\sqrt{3} - 4 - \frac{\pi}{3}$

(JEE Main 2014)

9. The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36-12x+x^2)}dx$ is equal to

(1) 2 (2) 4 (3) 1 (4) 6

(JEE Main 2015)

10. $\lim_{n\to\infty}\left(\frac{(n+1)(n+2)\dots 3n}{n^{2n}}\right)^{1/n}$ is equal to

(1) $\frac{27}{e^2}$ (2) $\frac{9}{e^2}$ (3) $3\log 3 - 2$ (4) $\frac{18}{e^4}$

(JEE Main 2016)

11. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{dx}{1+\cos x}$ is equal to

(1) -1 (2) -2 (3) 2 (4) 4

(JEE Main 2017)

12. The value of $\int_{-\pi/2}^{\pi/2}\frac{\sin^2 x}{1+2^x}dx$ is

(1) $\pi/4$ (2) $\pi/8$ (3) $\pi/2$ (4) 4π

(JEE Main 2018)

Single Correct Answer Type

1. (4) Let $I = \int_0^{\pi}[\cot x]\,dx$ ... (i)

$= \int_0^{\pi}[\cot(\pi - x)]\,dx = \int_0^{\pi}[-\cot x]\,dx$ ... (ii)

Adding (i) and (ii), we get

$$2I = \int_0^{\pi}[\cot x]\,dx + \int_0^{\pi}[-\cot x]\,dx = \int_0^{\pi}(-1)\,dx$$

[Since $[x] + [-x]$ is equal to -1 if $x \notin Z$, and is equal to 0 if $x \in Z$]

$= [-x]_0^{\pi} = -\pi$

$\therefore \quad I = -\dfrac{\pi}{2}$

2. (3) $p'(x) = p'(1 - x)$

$\Rightarrow \quad p(x) = -p(1 - x) + c$

At $x = 0$,

$p(0) = -p(1) + c \Rightarrow 42 = c$

$\therefore \quad p(x) = -p(1 - x) + 42$

$\Rightarrow \quad p(x) + p(1 - x) = 42$

$$I = \int_0^1 p(x)\,dx = \int_0^1 p(1 - x)\,dx$$

$$\therefore \quad 2I = \int_0^1 (p(x) + p(1 - x))dx = \int_0^1 (42)\,dx$$

$\Rightarrow \quad I = 21$

3. (2) We have

$$I = \int_0^1 \frac{8\log(1 + x)}{1 + x^2}dx$$

Put $x = \tan\theta$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} 8\cdot\frac{\log(1 + \tan\theta)}{\sec^2\theta}\sec^2\theta\,d\theta$$

$$= 8\int_0^{\pi/4}\log(1 + \tan\theta)d\theta$$

$$= 8\int_0^{\pi/4}\log(1 + \tan(\pi/4 - \theta))\,d\theta$$

$$= 8\int_0^{\pi/4}\log\left(1 + \frac{1 - \tan\theta}{1 + \tan\theta}\right)d\theta$$

$$= 8\int_0^{\pi/4}\log\left(\frac{2}{1 + \tan\theta}\right)d\theta$$

$= 8(\log 2)\cdot\dfrac{\pi}{4} - I$

$\Rightarrow \quad 2I = 2\pi\log 2$

$\Rightarrow \quad I = \pi\log 2$

4. (1) We have

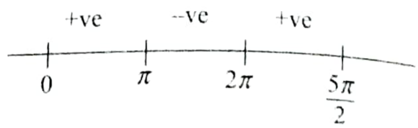
$$f(x) = \int_0^x \sqrt{t}\sin t\,dt$$

$\Rightarrow \quad f'(x) = \sqrt{x}\sin x$

For maximum of minimum value of $f(x)$, $f'(x) = 0$

$\Rightarrow \quad x = 2n\pi,\ n \in Z$

Sign scheme of $f'(x)$ for $x \in \left(0, \dfrac{5\pi}{2}\right)$ is



$f'(x)$ changes its sign from +ve to –ve in the neighborhood of π and from – to + in the neighborhood of 2π.

Hence $f(x)$ has local maximum at $x = \pi$ and local minima at $x = 2\pi$.

5. (2) $g(x + \pi) = \displaystyle\int_0^{x+\pi}\cos^4 t\,dt$

$$= \int_0^x \cos^4 t\,dt + \int_x^{x+\pi}\cos^4 t\,dt \qquad (\because \cos^4 t \text{ has period } \pi)$$

$$= g(x) + \int_0^{\pi}\cos^4 t\,dt$$

$= g(x) + g(\pi)$

6. (4) $I = \displaystyle\int_{\pi/6}^{\pi/3}\frac{dx}{1 + \sqrt{\tan x}}$ ... (i)

$$I = \int_{\pi/6}^{\pi/3}\frac{dx}{1 + \sqrt{\cot x}} = \int_{\pi/6}^{\pi/3}\frac{\sqrt{\tan x}\,dx}{\sqrt{\tan x} + 1} \quad \text{(ii)}$$

Adding (i) and (ii)

$$\Rightarrow \quad 2I = \int_{\pi/6}^{\pi/3} 1\,dx \Rightarrow 2I = \frac{\pi}{3} - \frac{\pi}{6}$$

$\Rightarrow \quad 2I = \dfrac{\pi}{6} \Rightarrow I = \dfrac{\pi}{12}$

7. (1) $y = \displaystyle\int_0^x |t|\,dt$

Case I : If $x > 0$

$$y = \int_0^x t\,dt = \left[\frac{t^2}{2}\right]_0^x = \frac{x^2}{2} \Rightarrow \frac{dy}{dx} = x = 2 \text{ (given)}$$

$\Rightarrow \quad x = 2$ and $y = 2$

$\therefore$ equation of tangent is $(y - 2) = 2(x - 2)$

or $\quad y - 2x + 2 = 0$

Hence, x intercept $= 1$.

Case II : $x < 0$

$$y = \int_0^x -t\,dt = \left[\frac{-t^2}{2}\right]_0^x = \frac{-x^2}{2}$$

$\therefore \quad \dfrac{dy}{dx} = -x = 2$

$\therefore \quad x = -2, \quad \therefore y = -2,$

∴ equation of tangent is $y + 2 = 2(x + 2)$

or $2x - y + 2 = 0$

∴ x intercept $= -1$

8. (4) $I = \int_0^{\pi} \sqrt{1 + 4\sin^2\frac{x}{2} - 4\sin\frac{x}{2}}\, dx$

$= \int_0^{\pi} \left|1 - 2\sin\frac{x}{2}\right| dx$

$= \int_0^{\pi/3} \left(1 - 2\sin\frac{x}{2}\right) dx + \int_{\pi/3}^{\pi} \left(2\sin\frac{x}{2} - 1\right) dx$

$= \left(x + 4\cos\frac{x}{2}\right)\Big|_0^{\pi/3} + \left(-4\cos\frac{x}{2} - x\right)\Big|_{\pi/3}^{\pi}$

$= 4\sqrt{3} - 4 - \frac{\pi}{3}$

9. (3) $I = \int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} dx$

$I = \frac{2}{2}\int_2^4 \frac{\log|x|}{\log|x| + \log|6 - x|} dx$ (i)

$I = \int_2^4 \frac{\log|6 - x|}{\log|6 - x| + \log|x|} dx \quad \left\{\int_a^b f(x)\,dx = \int_a^b f(a + b - x)\,dx\right\}$ (ii)

Adding (i) and (ii)

$2I = \int_2^4 \frac{\log|x| + \log|6 - x|}{\log|x| + \log|6 - x|} dx = \int_2^4 dx = 2$

Hence, $I = 1$

10. (1) $L = \lim_{n\to\infty} \left(\frac{(n+1)(n+2)\ldots\ldots(n+2n)}{n^{2n}}\right)^{1/n}$

∴ $\log_e L = \frac{1}{n}\left(\lim_{n\to\infty} \sum_{r=1}^{2n} \log\left(1 + \frac{r}{n}\right)\right)$

∴ $\log_e L = \int_0^2 \log(1 + x)\, dx$

∴ $\log_e L = (x\log(1 + x))_0^2 - \int_0^2 \frac{x}{1 + x} dx$

∴ $\log_e L = 2\log_e 3 - \int_0^2 \left(1 - \frac{1}{1 + x}\right) dx$

∴ $\log_e L = 2\log 3 - (x - \log(1 + x))_0^2$

∴ $\log_e L = 2\log 3 - (2 - \log 3)$

∴ $\log_e L = 3\log 3 - 2 = \log\frac{27}{e^2}$

∴ $L = \frac{27}{e^2}$

11. (3) $I = \int_{\pi/4}^{3\pi/4} \frac{dx}{1 + \cos x}$ ...(i)

$\Rightarrow \quad I = \int_{\pi/4}^{3\pi/4} \frac{dx}{1 - \cos x}$ ...(ii)

Adding (i) and (ii), we get

$2I = \int_{\pi/4}^{3\pi/4} \frac{2}{\sin^2 x} dx$

$\Rightarrow \quad I = \int_{\pi/4}^{3\pi/4} \text{cosec}^2 dx$

$\Rightarrow \quad I = -(\cot x)_{\pi/4}^{3\pi/4} = 2$

12. (1) $I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1 + 2^x} dx$ ...(i)

or $\quad I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2(-x)}{1 + 2^{-x}} dx$

or $\quad I = \int_{-\pi/2}^{\pi/2} \frac{2^x \cdot \sin^2 x}{2^x + 1} dx$ ...(ii)

Adding (i) and (ii), we get

$2I = \int_{-\pi/2}^{\pi/2} \sin^2 x\, dx = 2\int_0^{\pi/2} \sin^2 x\, dx$

$= \int_0^{\pi/2} (1 - \cos 2x) dx$

$= \left[x - \frac{\sin 2x}{2}\right]_0^{\pi/2} = \frac{\pi}{2}$

∴ $\quad I = \frac{\pi}{4}$

JEE ADVANCED

Single Correct Answer Type

1. (3) $f' = \pm\sqrt{1 - f^2}$

or $\quad f(x) = \sin x$ or $f'(x) = -\sin x$ (not possible)

∴ $\quad f(x) = \sin x$

Also, $x > \sin x \ \forall\ x > 0$.

2. (1) $\int_0^1 \left(x^6 - 4x^5 + 5x^4 - 4x^2 + 4 - \frac{4}{1 + x^2}\right) dx$

$= \left[\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x\right]_0^1 - \pi$

$= \frac{1}{7} - \frac{2}{3} + 1 - \frac{4}{3} + 4 - \pi = \frac{22}{7} - \pi$