Q
$$I = \int_{50}^{100} \frac{\ln x}{\ln x + \ln(150-x)}\, dx$$

Ans
$$I = \int_{50}^{100} \frac{\ln(150-x)}{\ln(150-x) + \ln x}\, dx$$

$$2I = \int_{50}^{100} dx \qquad I = 25$$

Q
$$\int_{0}^{\pi} \frac{2x \sin x \, dx}{3 + \cos 2x}$$

$$I = \int_{0}^{\pi} \frac{2(\pi - x)\sin x}{3 + \cos 2x}$$

$$2I = 2\int_{0}^{\pi} \frac{\cancel{\sin x}\ \pi \sin x}{3 + \cos 2x}$$

we get $\frac{\pi^2}{4}$

Q
$$\int_{0}^{2} \frac{dx}{(17 + 8x - 4x^2)(e^{6(1-x)} + 1)}$$

$I =$ after using
$$\int_{0}^{2} \frac{e^{6(1-x)}\, dx}{(17+8x-4x^2)(e^{6(1-x)}+1)}$$

$$2I = \int_{0}^{2} \frac{dx}{\text{Quad.}} \Rightarrow I = \frac{1}{2\sqrt{2}} \ln\left(\frac{\sqrt{21}-2}{\sqrt{21}+2}\right)$$

Date:

1. $\int_0^1 \cot^{-1}(1+x^2-x)\, dx$

Ans. $\int_0^1 \tan^{-1}\left(\frac{x+(1-x)}{1+x(1-x)}\right) dx$

$I = \int_0^1 (\tan^{-1}x + \tan^{-1}(1-x))\, dx$

{ indefinite main $x-1, 1-x$ --- $><0$ sochna nhi hai, definite main hai }

$2I = 2\int_0^1 \tan^{-1}x\, dx$

$I = \int_0^1 \tan^{-1}x\, dx$

$\Rightarrow \left(\frac{\pi}{4} - \ln 2\right)$

Q $\int_0^4 x(x-1)(x-2)(x-3)(x-4)\, dx$

Ans. $2I = \int 0$

$\underline{\underline{I = 0}}$

Q $\int_0^1 \frac{\ln(1+x)}{1+x^2} dx$

Ans. $x \to \tan\theta$

$\int_0^{\frac{\pi}{4}} \ln(1+\tan\theta)\, d\theta = \int_0^{\frac{\pi}{4}} \ln(\sin + \cos) - \int \ln(\cos\theta)\, d\theta$

P-6.

$$\int_{-a}^{a} f(x)\,dx = \int_{0}^{a} (f(x)+f(-x))\,dx = \begin{cases} 2\int_0^a f(x)dx & : \text{even} \\ 0 & : \text{odd} \end{cases}$$

Q $$\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\sec x}{|x|} \ln\left(\frac{1-x}{1+x}\right) dx$$

$= 0$

Q $$\int_{-1}^{3} \tan^{-1}\left(\frac{x}{x^2+1}\right) + \tan^{-1}\left(\frac{x^2+1}{x}\right) dx$$

Ans. $$\int_{-1}^{1} (\quad)\, dx + \int_{1}^{3} \left(\tan^{-1}\left(\frac{x}{x^2+1}\right) + \cot^{-1}\left(\frac{x}{x^2+1}\right)\right)dx$$

$$= \frac{\pi}{2}x\Big]_1^{3} \Rightarrow \underline{\underline{\pi}}.$$

Q $$\int_{-\frac{1}{\sqrt3}}^{\frac{1}{\sqrt3}} \frac{x^4}{1-x^4} \cos^{-1}\left(\frac{2x}{1+x^2}\right) dx$$

Ans.

King Queen.

$$I = \int_{-\frac{1}{\sqrt3}}^{\frac{1}{\sqrt3}} \frac{x^4}{1-x^4} \cos^{-1}\left(\frac{-2x}{1+x^2}\right) dx = \int_{-\frac{1}{\sqrt3}}^{\frac{1}{\sqrt3}} \frac{x^4\pi}{1-x^4} - \frac{x^4 \cos^{-1}\left(\frac{2x}{1+x^2}\right)}{1-x^4}$$

$$= \int_{-\frac{1}{\sqrt3}}^{\frac{1}{\sqrt3}} \frac{x^4\pi}{1-x^4} = \frac{\pi^2}{12} - \frac{\pi}{3} - \frac{\pi}{4}\ln(2-\sqrt3).$$

Q $\displaystyle\int_{-1}^{1} \frac{dx}{(x^2+x+1)+\sqrt{x^4+3x^2+1}}$

$$= \int_{-1}^{1} \frac{(x^2+x+1) - \sqrt{x^4+3x^2+1}}{2x(x^2+1)}$$

$$= \int_{-1}^{1} \frac{(x^2+x+1)dx}{2x(x^2+1)} = \int_{-1}^{1} \frac{dx}{2x} + \int_{-1}^{1} \frac{dx}{2(x^2+1)}$$

Q P-7.

(imp) (at last)

$$\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx$$

shifted even to $x=a$ → f(x) is symm. about 'a'

Also, $$\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^{a} f(x)\,dx & : f(2a-x) = f(x) \\ 0 & : f(2a-x) = -f(x) \end{cases}$$

shifted odd to $x=a$

eg:- $$\int_0^{2\pi} \cos^7 x\,dx$$

$$= ~~2\int_0^{2.(\pi)/2} \cos^7 x = 0~~$$

~~odd at π~~

$$= 2.\int_0^{2.\frac{\pi}{2}} \cos^7 x\,dx = 0$$

odd at $\frac{\pi}{2}$