## Problem 2:

**Example 8 (Manufacturing problem)** A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realise a maximum profit? What is the maximum profit per week?

**Solution** Suppose $x$ is the number of pieces of Model A and $y$ is the number of pieces of Model B. Then

$$\text{Total profit (in Rs)} = 8000\,x + 12000\,y$$

Let $$Z = 8000\,x + 12000\,y$$

We now have the following mathematical model for the given problem.

Maximise $Z = 8000\,x + 12000\,y$ ... (1)

subject to the constraints:

$9x + 12y \le 180$ (Fabricating constraint)

i.e. $3x + 4y \le 60$ ... (2)

$x + 3y \le 30$ (Finishing constraint) ... (3)

$x \ge 0,\ y \ge 0$ (non-negative constraint) ... (4)

The feasible region (shaded) OABC determined by the linear inequalities (2) to (4) is shown in the Fig 12.9. Note that the feasible region is bounded.

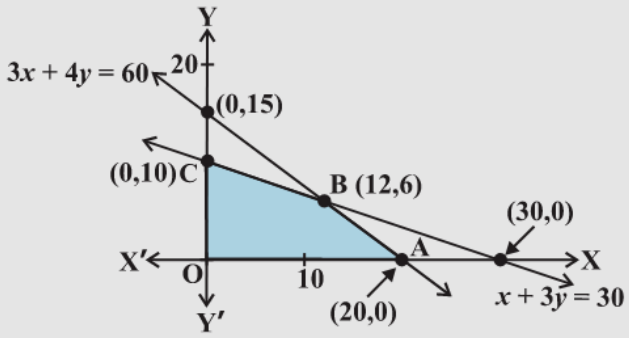


**Fig 12.9**

Let us evaluate the objective function Z at each corner point as shown below:

| Corner Point | $Z = 8000\,x + 12000\,y$ | |
|---|---|---|
| 0 (0, 0) | 0 | |
| A (20, 0) | 160000 | |
| B (12, 6) | **168000** ← | Maximum |
| C (0, 10) | 120000 | |

We find that maximum value of Z is 1,68,000 at B (12, 6). Hence, the company should produce 12 pieces of Model A and 6 pieces of Model B to realise maximum profit and maximum profit then will be Rs 1,68,000.