Definite Integration

$$\int_a^b f(x)\,dx = (g(x))_a^b = g(b) - g(a)$$

i) continous on (a,b)

ii) $\left.\begin{array}{l} g(b^-) - g(a) \\ g(b) - g(a^+) \\ g(b^-) - g(a^+) \end{array}\right\}$ if $f(x)$ is not defined

eg:- $\int_0^1 \ln x\,dx = (x\ln x - x)_0^1 = -1 - \lim_{x\to 0^+} x\ln x$

$= -1 - 0 = -1$

Q $\int_{\frac{1}{2}}^{2} \left(1 + x - \frac{1}{x}\right) e^{x+\frac{1}{x}}\,dx$

Ans $\int_{\frac{1}{2}}^{2} \Big(\underbrace{e^{x+\frac{1}{x}}}_{f(x)} + x f'(x)\Big)\,dx$

$= \left(x e^{x+\frac{1}{x}}\right)_{1/2}^{2}$

Q $\int_3^5 \frac{x^2\,dx}{\sqrt{(x-3)(5-x)}}$

Ans $x = 3\cos^2\theta + 5\sin^2\theta$

$2\int_3^5 (9 + 4\sin^4\theta + 12\sin^2\theta)\,d\theta \Rightarrow$ we get $\frac{33\pi}{2}$

Properties

① $\displaystyle\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

② $\displaystyle\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

③ $\displaystyle\int_a^b f(x)\,dx = \underbrace{\int_a^c f(x)\,dx}_{\text{continous in } \boxed{a \to c}} + \underbrace{\int_c^b f(x)\,dx}_{\text{continous in } \boxed{c \to b}}$

{ c may not necessarily be in between a to b as $-\int_c^b f(x) = +\int_b^c f(x)$ }

* $\displaystyle\int_a^b f(x)\,dx = \int_{a+c}^{b+c} f(x-c)\,dx \quad : c > 0$

* $\displaystyle K\int_{a/K}^{b/K} f(Kx)\,dx = \int_a^b f(x)\,dx$

eg :- $\displaystyle\int_0^{\pi/2} f(\sin 2x) = \frac{1}{2}\int_0^{\pi/4} f(\sin x)$

here $K = \frac{1}{2}$

Q $\displaystyle\int_{\frac{1}{2}}^{2} \frac{1}{x}\,\text{cosec}^{101}\left(x - \frac{1}{x}\right) dx$

Ans $x \to \frac{1}{t}$

$\displaystyle\int_2^{\frac{1}{2}} t\,\text{cosec}^{101}\left(\frac{1}{t} - t\right)\left(\frac{-1}{t^2}\right) dt$

$$= -\int_{\frac{1}{2}}^{2} \frac{1}{x} \operatorname{cosec}^{101}\left(x - \frac{1}{x}\right)$$

$$I = -I$$

$$\Downarrow$$

$$\boxed{I = 0}$$

Q $\displaystyle\int_0^\infty \frac{\ln x}{1+x^2}\,dx$

Ans. $x \to \frac{1}{t}$ $\displaystyle\int_\infty^0 \frac{(-\ln t)t^2}{1+t^2} \times \frac{(-dt)}{t^2} = -\int_0^\infty \frac{\ln x\,dx}{1+x^2}$

$$I = -I$$

$$I = 0$$

Q $\displaystyle\int_0^\infty \frac{\tan^{-1}x}{x^2-x+1}\,dx$

Ans. $x = \frac{1}{t}$ $\quad dx = \frac{-1}{t^2}\,dt$

$$I = \int_\infty^0 \frac{-\cot^{-1}(t)}{t^2-t+1}\,dt$$

$$I + I = \int_0^\infty \frac{\pi}{2(t^2-t+1)}$$

Q $\int_0^\infty \frac{dx}{(1+x^2)(1+x^{17})}$

$x = \frac{1}{t}$

$I = \int_0^\infty \frac{x^{17}\,dx}{(1+x^2)(1+x^{17})}$

$2I = \int_0^\infty \frac{dx}{1+x^2}$

$2I = \frac{\pi}{2} \qquad I = \frac{\pi}{4}$

By Parts in Definite Integration

* $\int_a^b v.u\,dx = u \int v\,dx \Big|_a^b - \int_a^b \left(\left(u' \int v\,dx\right) dx\right)$

* ya fir whole pe hi limit lga de!

Q $\dfrac{\int_0^{\frac{\pi}{2}} (\cos x)^{\sqrt{2}+1}\,dx}{\int_0^{\frac{\pi}{2}} (\cos x)^{\sqrt{2}-1}\,dx}$

Ans $I_1 = \int_0^{\frac{\pi}{2}} \underset{v}{(\cos x)}\,\underset{u}{(\cos x)^{\sqrt{2}}} = \left((\cos x)^{\sqrt{2}} (\sin x)\right)_0^{\frac{\pi}{2}}$

$+ \sqrt{2} \int_0^{\frac{\pi}{2}} (\cos x)^{\sqrt{2}-1} (\sin^2 x)\,dx$

$1-\cos^2 x$

$$\int_0^{\frac{\pi}{2}} (\cos x)^{\sqrt{2}+1} = \sqrt{2}\int_0^{\frac{\pi}{2}} (\cos x)^{\sqrt{2}-1} - \sqrt{2}\int_0^{\frac{\pi}{2}} (\cos x)^{\sqrt{2}+1}$$

Ans $\Rightarrow$ $\dfrac{\sqrt{2}}{\sqrt{2}+1}$

Q $$\frac{\int_0^1 \left(1-(1-x^3)^{\sqrt{2}}\right)^{\sqrt{3}} x^2\,dx}{\int_0^1 \left(1-(1-x^3)^{\sqrt{2}}\right)^{\sqrt{3}+1} x^2\,dx}$$

$1-x^3 = t$

$$= \frac{\int_0^1 (1-t^{\sqrt{2}})^{\sqrt{3}}\,dt \rightarrow I_1}{\int_0^1 (1-t^{\sqrt{2}})^{\sqrt{3}+1}\,dt \rightarrow I_2}$$

$$I_2 = \int_0^1 \underbrace{1}_{v}\cdot\underbrace{(1-t^{\sqrt{2}})^{\sqrt{3}+1}}_{u}\,dt$$

$$= (1-t^{\sqrt{2}})^{\sqrt{3}+1}\, t\Big|_0^1 + \int_0^1 (\sqrt{3}+1)(1-t^{\sqrt{2}})^{\sqrt{3}}\,\sqrt{2}\,(t^{\sqrt{2}-1})\,t\,dt$$

$$\int_0^1 (\sqrt{3}+1)\sqrt{2}\,(1-t^{\sqrt{2}})^{\sqrt{3}}(1-t^{\sqrt{2}}-1)\,dt$$

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$\int_{-x}^{x} \rightarrow 2\int^{x}$

odd = 0
even = even

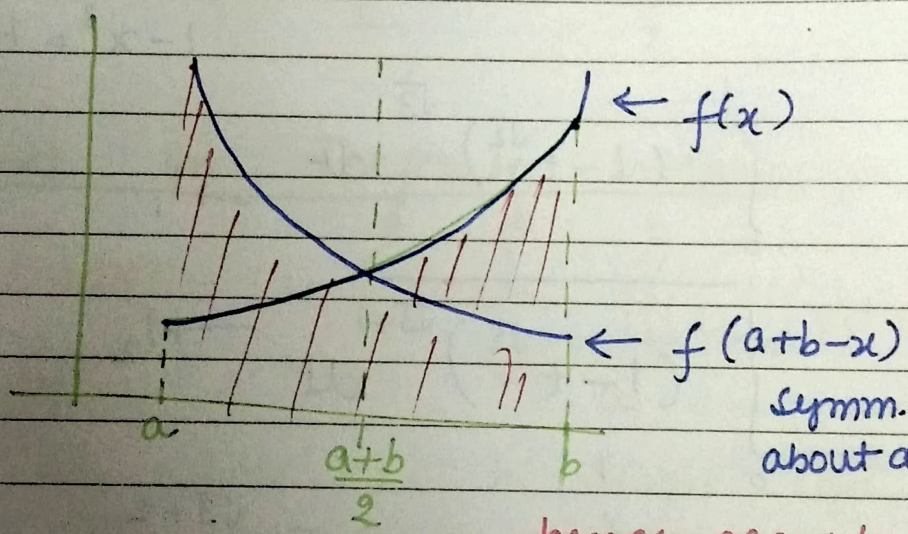
④ King

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

⑤ Queen

$$\int_a^b f(x)\cdot dx = \int_a^b f(a+b-x)\,dx$$

Proof



symm. about $\frac{a+b}{2}$

hence proved

Q $$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$$

King

$$2I = \int_0^{\frac{\pi}{2}} 1\,dx \Rightarrow I = \frac{\pi}{4}$$

Q $I = \int_{50}^{100} \frac{\ln x}{\ln x + \ln(150-x)} dx$

Ans $I = \int_{50}^{100} \frac{\ln(150-x)}{\ln(150-x) + \ln x} dx$

$2I = \int_{50}^{100} dx \qquad I = 25$

Q $\int_{0}^{\pi} \frac{2x \sin x \, dx}{3 + \cos 2x}$

$I = \int_{0}^{\pi} \frac{2(\pi - x) \sin x}{3 + \cos 2x}$

$2I = 2\int_{0}^{\pi} \frac{\pi \sin x}{3 + \cos 2x}$

we get $\frac{\pi^2}{4}$

Q $\int_{0}^{2} \frac{dx}{(17 + 8x - 4x^2)(e^{6(1-x)} + 1)}$

$I =$ after king $\int_{0}^{2} \frac{e^{6(1-x)} \, dx}{(17+8x-4x^2)(e^{6(1-x)}+1)}$

$2I = \int_{0}^{2} \frac{dx}{\text{Quad.}} \Rightarrow I = \frac{1}{2\sqrt{2}} \ln\left(\frac{\sqrt{21}-2}{\sqrt{21}+2}\right)$