

$$\mathbf{R} \frac{|1 - z_1| |1 - z_2| \dots |1 - z_9|}{10} \text{ equals}$$

(R) \rightarrow (3) : We know $z^{10} - 1 = (z - 1)(z - z_1) \dots (z - z_9)$

$$\Rightarrow (z - z_1)(z - z_2) \dots (z - z_9) = \frac{z^{10} - 1}{z - 1}$$

$$= 1 + z + z^2 + \dots + z^9$$

For $z = 1$ we get

$$(1 - z_1)(1 - z_2) \dots (1 - z_9) = 10$$

$$\therefore \frac{|1 - z_1| |1 - z_2| \dots |1 - z_9|}{10} = 1$$

S. $1 - \sum_{k=1}^9 \cos\left(\frac{2k\pi}{10}\right)$ equals

(S) \rightarrow (4) : $1, Z_1, Z_2, \dots, Z_9$ are 10th roots of unity.

$$\therefore Z^{10} - 1 = 0$$

From equation $1 + Z_1 + Z_2 + \dots + Z_9 = 0$

$$\operatorname{Re}(1) + \operatorname{Re}(Z_1) + \operatorname{Re}(Z_2) + \dots + \operatorname{Re}(Z_9) = 0$$

$$\Rightarrow \operatorname{Re}(Z_1) + \operatorname{Re}(Z_2) + \dots + \operatorname{Re}(Z_9) = -1$$

$$\Rightarrow \sum_{K=1}^9 \cos \frac{2k\pi}{10} = -1 \Rightarrow 1 - \sum_{K=1}^9 \cos \frac{2k\pi}{10} = 2$$

Hence (c) is the correct option.

13. The value of $\sum_{k=1}^{10} \left(\sin \frac{2k\pi}{11} + i \cos \frac{2k\pi}{11} \right)$ is [2006]

- (a) i (b) 1 (c) -1 (d) $-i$

13. (d)

$$\sum_{k=1}^{10} \left(\sin \frac{2k\pi}{11} + i \cos \frac{2k\pi}{11} \right) = i \sum_{k=1}^{10} \left(\cos \frac{2k\pi}{11} - i \sin \frac{2k\pi}{11} \right)$$

$$= i \sum_{k=1}^{10} e^{-\frac{2k\pi}{11}i} = i \left\{ \sum_{k=0}^{10} e^{-\frac{2k\pi}{11}i} - 1 \right\}$$

$$= i \left[1 + e^{-\frac{2\pi}{11}i} + e^{-\frac{4\pi}{11}i} + \dots + 11 \text{ terms} \right] - i$$

$$= i \left[\frac{1 - \left(e^{-\frac{2\pi}{11}i} \right)^{11}}{1 - e^{-\frac{2\pi}{11}i}} \right] - i = i \left[\frac{1 - e^{-2\pi i}}{1 - e^{-\frac{2\pi}{11}i}} \right] - i$$

$$= i \times 0 - i \quad [\because e^{-2\pi i} = 1] = -i$$