

5. Suppose Z_1, Z_2, Z_3 are the vertices of an equilateral triangle inscribed in the circle $|Z| = 2$. If $Z_1 = 1 + i\sqrt{3}$ then $Z_2 = \dots\dots\dots$,
 $Z_3 = \dots\dots\dots$ *(1994 - 2 Marks)*

6. The value of the expression
 $1 \cdot (2 - \omega)(2 - \omega^2) + 2 \cdot (3 - \omega)(3 - \omega^2) + \dots + (n - 1) \cdot (n - \omega)(n - \omega^2)$,
where ω is an imaginary cube root of unity, is.....
(1996 - 2 Marks)

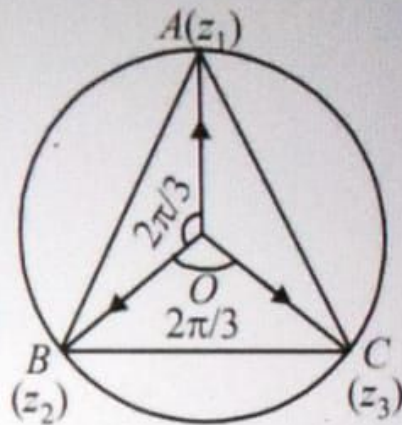
5. Let z_1, z_2, z_3 be the vertices A, B and C respectively of equilateral ΔABC , inscribed in a circle $|z| = 2$, centre $(0, 0)$ radius = 2

Given $z_1 = 1 + i\sqrt{3}$

$$z_2 = e^{\frac{2\pi i}{3}} z_1$$

$$= \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right) (1 + i\sqrt{3})$$

$$= \frac{-1 - 3}{2} = -2$$



and $z_3 = e^{4(\pi/3)i} z_1$

$$= \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right) (1 + i\sqrt{3})$$

$$= \left(\frac{-1 - i\sqrt{3}}{2} \right) (1 + i\sqrt{3}) = \frac{-1 - 2i\sqrt{3} + 3}{2} = 1 - i\sqrt{3}$$

6. r th term of the given series,

$$= r[(r+1) - \omega](r+1) - \omega^2]$$

$$= r[(r+1)^2 - (\omega + \omega^2)(r+1) + \omega^3]$$

$$= r[(r+1)^2 - (-1)(r+1) + 1]$$

$$= r[(r^2 + 3r + 3)] = r^3 + 3r^2 + 3r$$

Thus, sum of the given series,

$$= \sum_{r=1}^{(n-1)} (r^3 + 3r^2 + 3r)$$

$$= \frac{1}{4}(n-1)^2 n^2 + 3 \cdot \frac{1}{6}(n-1)(n)(2n-1) + 3 \cdot \frac{1}{2}(n-1)n$$

$$= (n-1)(n) \left[\frac{1}{4}(n-1)n + \frac{1}{2}(2n-1) + \frac{3}{2} \right]$$

$$= \frac{1}{4}(n-1)n[n^2 - n + 4n - 2 + 6]$$

$$= \frac{1}{4}(n-1)n[n^2 + 3n + 4]$$

2. If the complex numbers, Z_1 , Z_2 and Z_3 represent the vertices of an equilateral triangle such that $|Z_1| = |Z_2| = |Z_3|$ then $Z_1 + Z_2 + Z_3 = 0$. *(1984 - 1 Mark)*

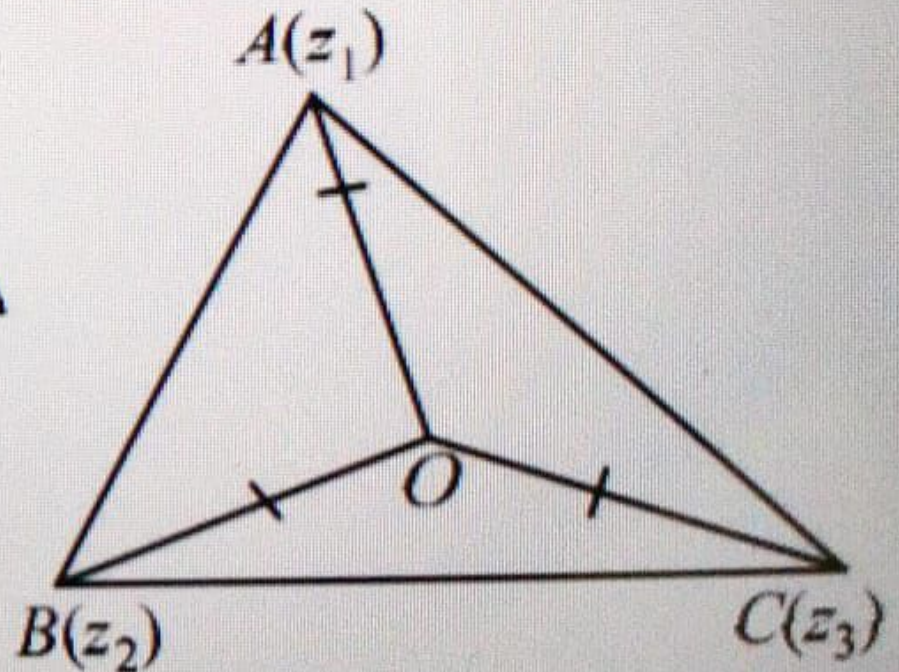
2. As $|z_1| = |z_2| = |z_3|$
 $\therefore z_1, z_2, z_3$ are equidistant from origin. Hence O is the circumcentre of ΔABC .

But according to question ΔABC is equilateral and we know that in an equilateral Δ circumcentre and centroid coincide.

\therefore Centroid of $\Delta ABC = 0$

$$\Rightarrow \frac{z_1 + z_2 + z_3}{3} = 0 \Rightarrow z_1 + z_2 + z_3 = 0$$

\therefore Statement is true.



4. The cube roots of unity when represented on Argand diagram form the vertices of an equilateral triangle.

(1988 - 1 Mark)

4. \therefore Cube roots of unity are $1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$

\therefore Vertices of triangle are

$$A(1, 0), B\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right), C\left(\frac{-1}{2}, \frac{-\sqrt{3}}{2}\right)$$

$\Rightarrow AB = BC = CA \quad \therefore \Delta$ is equilateral.

1. If the cube roots of unity are $1, \omega, \omega^2$, then the roots of the equation $(x - 1)^3 + 8 = 0$ are (1979)

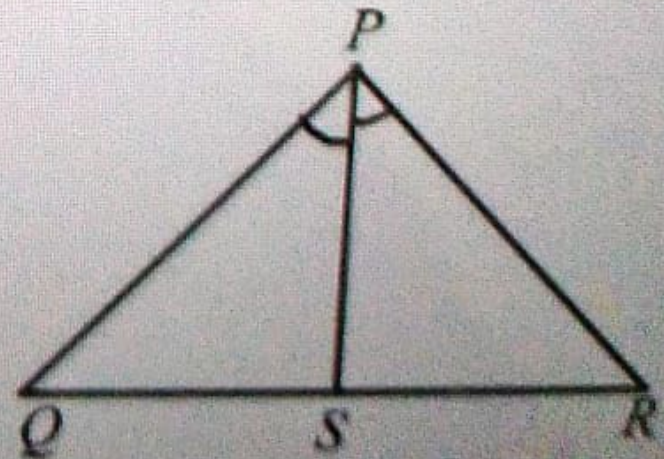
(a) $-1, 1 + 2\omega, 1 + 2\omega^2$

(b) $-1, 1 - 2\omega, 1 - 2\omega^2$

(c) $-1, -1, -1$

(d) None of these

1. (b) $(x-1)^3 + 8 = 0$
 $\Rightarrow (x-1)^3 = -8 = (-2)^3$
 $\Rightarrow x-1 = -2$
or -2ω or $-2\omega^2$
 $\Rightarrow x = -1, 1-2\omega, 1-2\omega^2$



9. If $\omega (\neq 1)$ is a cube root of unity and $(1 + \omega)^7 = A + B\omega$ then A and B are respectively (1995S)
- (a) 0, 1 (b) 1, 1 (c) 1, 0 (d) -1, 1

$$9. \quad (b) \quad (1+\omega)^7 = A + B\omega$$

$$\Rightarrow (-\omega^2)^7 = A + B\omega \quad (\because 1 + \omega + \omega^2 = 0)$$

$$\Rightarrow -\omega^{14} = A + B\omega$$

$$\Rightarrow -\omega^2 = A + B\omega \quad (\because \omega^3 = 1)$$

$$\Rightarrow 1 + \omega = A + B\omega \Rightarrow A = 1, B = 1$$

13. If $i = \sqrt{-1}$, then $4 + 5 \left(-\frac{1}{2} + \frac{i\sqrt{3}}{2} \right)^{334} + 3 \left(-\frac{1}{2} + \frac{i\sqrt{3}}{2} \right)^{365}$

is equal to

(1999 - 2 Marks)

- (a) $1 - i\sqrt{3}$ (b) $-1 + i\sqrt{3}$ (c) $i\sqrt{3}$ (d) $-i\sqrt{3}$

$$20. \quad (b) \quad (1 + \omega^2)^n = (1 + \omega^4)^n$$

$$\Rightarrow (-\omega)^n = (1 + \omega)^n = (-\omega^2)^n \Rightarrow \omega^n = 1 \Rightarrow n = 3$$

20. If $\omega (\neq 1)$ be a cube root of unity and $(1 + \omega^2)^n = (1 + \omega^4)^n$,
then the least positive value of n is (2004S)

- (a) 2 (b) 3 (c) 5 (d) 6

$$20. \quad (b) \quad (1 + \omega^2)^n = (1 + \omega^4)^n$$

$$\Rightarrow (-\omega)^n = (1 + \omega)^n = (-\omega^2)^n \Rightarrow \omega^n = 1 \Rightarrow n = 3$$