6. If $\left(\frac{1+i}{1-i}\right)^x = 1$ then [2003]

(a) $x = 2n+1$, where n is any positive integer

(b) $x = 4n$, where n is any positive integer

(c) $x = 2n$, where n is any positive integer

(d) $x = 4n+1$, where n is any positive integer.

6. **(b)** $\left(\frac{1+i}{1-i}\right)^x = 1 \Rightarrow \left[\frac{(1+i)^2}{1-i^2}\right]^x = 1$

$$\left(\frac{1+i^2+2i}{1+1}\right)^x = 1 \Rightarrow (i)^x = 1; \therefore x = 4n; \quad n \in I^+$$

8. If $z = x - i\,y$ and $z^{\frac{1}{3}} = p + iq$, then $\left(\frac{x}{p} + \frac{y}{q}\right) \Big/ (p^2 + q^2)$ is equal to [2004]

(a) -2 (b) -1 (c) 2 (d) 1

[2004]

8. **(a)** $z^{\frac{1}{3}} = p + iq \Rightarrow z = p^3 + (iq)^3 + 3p(iq)(p + iq)$

$\Rightarrow x - iy = p^3 - 3pq^2 + i(3p^2q - q^3)$

$\therefore x = p^3 - 3pq^2 \Rightarrow \frac{x}{p} = p^2 - 3q^2$

$y = q^3 - 3p^2q \Rightarrow \frac{y}{q} = q^2 - 3p^2$

$\therefore \frac{x}{p} + \frac{y}{q} = -2p^2 - 2q^2 \;\; \therefore \left(\frac{x}{p} + \frac{y}{q}\right) \Big/ (p^2 + q^2) = -2$

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Problem 11)

If the expression

$$\frac{\left[\sin\left(\frac{x}{2}\right) + \cos(x/2) + i\tan(x)\right]}{\left[1 + 2i\sin(x/2)\right]}$$

is real, then the set of all possible values of x is -

Solution) Let $z = \dfrac{\sin x/2 + \cos x/2 + i\tan x}{1 + 2i\sin x/2}$

$$\Rightarrow \frac{(\sin x/2 + \cos x/2 + i\tan x)(1 - 2i\sin x/2)}{(1 + 2i\sin x/2)(1 - 2i\sin x/2)}$$

After finding real part and Imaginary part

put $Im(z) = 0$ (as z is real)

$$\tan x - 2\sin\frac{x}{2}\left(\sin x/2 + \cos x/2\right) = 0$$

$$\frac{\sin x}{\cos x} - 2\sin^2 x/2 - 2\sin x/2 \cos x/2 = 0$$

$$\Rightarrow \frac{\sin x}{\cos x} - (1 - \cos x) - \sin x = 0 .$$

$$\sin x\left[\frac{1}{\cos x} - 1\right] - [1 - \cos x] = 0$$

$$\left(\frac{1 - \cos x}{\cos x}\right)\sin x - [1 - \cos x] = 0$$

$$(1 - \cos x)\left(\frac{\sin x}{\cos x} - 1\right) = 0$$

$\cos x = 1 \Rightarrow x = 2n\pi$

$\tan x = 1 \quad x = n\pi + \pi/4 \quad \therefore x = 2n\pi, n\pi + \pi/4$

Problem 2) If a, b, c, are the numbers between 0 and 1 such that the points $z_1 = a+i$, $z_2 = 1+bi$ and $z_3 = 0$ form an equilateral triangle then find a & b.

Solution) $|z_1 - z_2|$ = distance b/w two points represented by z_1 and z_2.

As $z_1 = a+i$, $z_2 = 1+bi$ and $z_3 = 0$ form an equilateral triangle, therefore.

$|z_1 - z_3| = |z_2 - z_3| = |z_1 - z_2|$

$|a+i| = |1+bi| = |(a-1)+i(1-b)|$

$a^2+1 = 1+b^2 = (a-1)^2+(1-b)^2$

$a^2 = b^2 = a^2+b^2-2a-2b+1$

$\Rightarrow a = b$

$\because a, b > 0 \quad \therefore \quad a \neq -b$ s

$b^2 - 2a - 2b + 1 = 0$

$a^2 - 2a - 2b + 1 = 0$

$a^2 - 2a - 2a + 1 = 0 \qquad [\because a = b]$

$a^2 - 4a + 1 = 0$

$a = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$ but $0 < a, b < 1$

$\therefore \quad a = 2-\sqrt{3}$ also $b = 2-\sqrt{3}$.