

Example 13 If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

Solution We have $x^y = e^{x-y}$. Taking logarithm on both sides, we get

$$y \log x = x - y$$

$$\Rightarrow y(1 + \log x) = x$$

$$\text{i.e. } y = \frac{x}{1 + \log x}$$

Differentiating both sides w.r.t. x , we get

$$\frac{dy}{dx} = \frac{(1 + \log x) \cdot 1 - x \left(\frac{1}{x} \right)}{(1 + \log x)^2} = \frac{\log x}{(1 + \log x)^2}.$$