Question 18. Let e denote the base of the natural logarithm. The value of the real number a for which the right-hand limit is equal to a nonzero real number, is _____

Solution:

Answer: 1.00

$$L=\lim_{x\to 0^+}\frac{(1-x)^{\frac{1}{x}}-e^{-1}}{x^a}$$

$$L=\lim_{x\to 0^+}\frac{e^{\ln(1-x)^{1/x}}-e^{-1}}{x^a}$$

$$L=\lim_{x\to 0^+}\frac{e^{\frac{1}{x}\ln(1-x)}-e^{-1}}{x^a}$$

$$L=\lim_{x\to 0^+}\frac{e^{\frac{1}{x}\left(-x-\frac{x^2}{2}-\frac{x^3}{3}\ldots\right)}-e^{-1}}{x^a}$$

$$L=\lim_{x\to 0^+}\frac{e^{-1}.e^{-\left(\frac{x}{2}+\frac{x^2}{3}\ldots\right)}-e^{-1}}{x^a}$$

$$L=\lim_{x\to 0^+}\frac{e^{-1}\left[e^{-\left(\frac{x}{2}+\frac{x^2}{3}\ldots\right)}-1\right]}{x^a}$$

$$L=\lim_{x\to 0^+}\frac{e^{-1}\left[\left(1+\left(-\frac{x}{2}-\frac{x^2}{3}\right)+\frac{\left(\frac{x}{2}+\frac{x^2}{3}\right)^2}{2!}\ldots\right)-1\right]}{x^a}$$

$$=\lim_{x\to 0^+}\frac{e^{-1}\left(\left(-\frac{1}{2}-\frac{x}{3}\ldots\right)+\frac{x\left(\frac{1}{2}+\frac{x}{3}\right)^2}{2!}\ldots\right)}{x^{a-1}}$$

For Non - Zero limit $a-1=0\Rightarrow a=1$