Question 13. Let m be the minimum possible value of $log_3(3^{y_1} + 3^{y_2} + 3^{y_3})$, where y_1, y_2, y_3 are real numbers for which $y_1 + y_2 + y_3 = 9$. Let M be the maximum possible value of $(\log_3 x_1 + \log_3 x_2 + \log_3 x_3)$, where x_1, x_2, x_3 are positive real numbers for which $x_1 + x_2 + x_3 = 9$. Then the value of $\log_2(m^3) + \log_3(M^2)$ is _____

Solution:

Answer: 8.00

Using AM ≥ GM

$\frac{3^{y_1}+3^{y_2}+3^{y_3}}{3} \geq (3^{y_1} \cdot 3^{y_2} \cdot 3^{y_3})^{\frac{1}{3}}$

$3^{y_1} + 3^{y_2} + 3^{y_3} \geq 3 \cdot (3^{y_1+y_2+y_3})^{\frac{1}{3}}$ $\{\therefore y_1 + y_2 + y_3 = 9\}$

$3^{y_1} + 3^{y_2} + 3^{y_3} \geq 3 \cdot (3^9)^{\frac{1}{3}}$

$3^{y_1} + 3^{y_2} + 3^{y_3} \geq 81$

$m = \log_3 81 = \log_3 3^4 = 4\log_3 3 = 4$

Again, using A.M ≥ G.M

$\frac{x_1+x_2+x_3}{3} \geq (x_1 \cdot x_2 \cdot x_3)^{\frac{1}{3}}$

$= \frac{9}{3} \geq (x_1 \cdot x_2 \cdot x_3)^{\frac{1}{3}}$ $\{\because x_1+x_2+x_3 = 9\}$

$\Rightarrow 27 \geq x_1x_2x_3$

$M = \log_3 x_1 + \log_3 x_2 + \log_3 x_3$

$M = \log_3(x_1x_2x_3) = \log_3(27) = 3$

$\therefore \log_2(m)^3 + \log_3(M)^2 \Rightarrow \log_2(2^6) + \log_3(3^2) = 6 + 2 = 8$