Indefinite Integration

Let $\frac{d}{dx}F(x) = f(x)$ $\frac{d}{dx}(F(x)+d) = f(x)$

$\therefore \int f(x)\cdot dx = F(x) + C$ [where C is called constant of integration]

→ Integration ⇒ Antiderivative, primitive

Properties of integration

① $\int a f(x)\cdot dx = a\int f(x)\cdot dx$ where a is constant

② $\int (K_1 f(x) \pm K_2 g(x))\cdot dx = K_1\int f(x)\cdot dx \pm K_2\int g(x)$
K_1, K_2 are constant

Integration of some standard functions

1. $\int \frac{1}{\sqrt{a^2-x^2}}\cdot dx = \sin^{-1}\left(\frac{x}{a}\right) + C = -\cos^{-1}\frac{x}{a} + C$

2. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C = -\frac{1}{a}\cot^{-1}\frac{x}{a} + C$

3. $\int \frac{1}{|x|\sqrt{x^2-a^2}}\cdot dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C = -\frac{1}{a}\operatorname{cosec}^{-1}\frac{x}{a} + C$

4. $\int \cot x\cdot dx = \int \frac{\cos x}{\sin x}\cdot dx = \ln|\sin x| + C = -\ln|\operatorname{cosec} x| + C$

5. $\int \tan x\cdot dx = -\int \frac{-\sin x}{\cos x}\cdot dx = -\ln|\cos x| + C$
$= \ln|\sec x| + C$

6. $\int \sec x \, dx = \int \frac{\sec x(\sec x + \tan x)}{(\sec x + \tan x)} \, dx = \ln|\sec x + \tan x| + c = \boxed{\ln\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + c}$

$= \boxed{-\ln|(\sec x - \tan x)| + c}$

~~$= -\ln|(\cot x + \operatorname{cosec} x)| + c$~~

7 $\int \operatorname{cosec} x \, dx = \int \frac{\operatorname{cosec} x(\cot x - \operatorname{cosec} x)}{(\cot x - \operatorname{cosec} x)} \, dx = \ln|(\cot x - \operatorname{cosec} x)| + c$

$= \ln\left|\tan \frac{x}{2}\right| + c$

$= -\ln|(\cot x + \operatorname{cosec} x)| + c$

8 $\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left|x + \sqrt{x^2 + a^2}\right| + c$

9 $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left|x + \sqrt{x^2 - a^2}\right| + c$

10 $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left|\frac{x - a}{x + a}\right| + c$

11 $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left|\frac{a + x}{a - x}\right| + c$

12 $\int \sqrt{u^2 + a^2} \, du = \frac{u}{2}\sqrt{u^2 + a^2} + \frac{a^2}{2} \ln\left|u + \sqrt{u^2 + a^2}\right| + c$

13 $\int \sqrt{u^2 - a^2} \, du = \frac{u}{2}\sqrt{u^2 - a^2} - \frac{a^2}{2} \ln\left|u + \sqrt{u^2 - a^2}\right| + c$

14 $\int \sqrt{a^2 - x^2} \, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\frac{x}{a} + c$

15 $\int e^{ax}\sin bx \, dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + c$

$$= \frac{e^{ax}}{\sqrt{a^2+b^2}}\sin\left(bx - \tan^{-1}\frac{b}{a}\right) + c$$

16 $\int e^{ax}\cos bx \, dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + c$

$$= \frac{e^{ax}}{\sqrt{a^2+b^2}}\cos\left(bx - \tan^{-1}\frac{b}{a}\right) + c$$

→ Let $\int f(x) \cdot dx = F(x) + c$ then

$$\boxed{\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + c}$$

a, b are constant

Eg- $\int \frac{dx}{\sqrt{a^2-(cx+d)^2}} = \boxed{\frac{1}{c}\sin^{-1}\frac{(cx+d)}{a} + \lambda}$

Eg- ① $\int \tan^2 x \, dx = \int (\sec^2 x - 1)\, dx = \tan x - x + c$

② $\int \sin^2 x \, dx = \int \frac{1-\cos 2x}{2}\, dx$

$$= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + c$$

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(3) $\int \frac{dx}{\sqrt{x+1}+\sqrt{x}} = \int (\sqrt{x+1}-\sqrt{x}).dx = \boxed{\frac{(x+1)^{3/2}}{3/2} - \frac{x^{3/2}}{\frac{3}{2}} + c}$

for integration use
- Standard formulas
- Changing to easy function
- Rationalisation
- for $\sin x$ give $1 = \sin^2 x + \cos^2 x$ use this

(4) $\int \sqrt{1-\sin 2x}\,.dx$

$\int \sqrt{(\sin^2 x + \cos^2 x - 2\sin x\cos x)}.dx = \int (\sin x - \cos x).dx$

$\int (\sin x - \cos x).dx$
$= \boxed{-\cos x - \sin x + c}$

$-\int (\sin x - \cos x).dx$
$= \boxed{-(-\sin x - \cos x) + c}$

* for this chapter (for general questions) → we don't use modulus here

$\int \sqrt{x^2}.dx = \int x.dx = \boxed{\frac{x^2}{2} + c}$

→ $\int \frac{1}{x}.dx = \ln|x| \Rightarrow$ general given $\boxed{\ln x + c}$

17. $\int \sin x.dx = -\cos x + c$

18. $\int \sec^2 x.dx = \tan x + c$

19. $\int \operatorname{cosec}^2 x.dx = -\cot x + c$

Ex. $\int \frac{dx}{\sin^2 x \cos^2 x} \longrightarrow \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}$

$= \int \sec^2 x + \csc^2 x = \boxed{\tan x - \cot x + C}$

$\int \frac{4}{\sin^2 2x} \cdot dx = 4 \int \csc^2 2x \cdot dx = \boxed{-\frac{4 \cot 2x}{2} + C}$

$= \boxed{-2 \cot 2x + C}$

Question

Q.1 $\int \frac{\cos 2x}{\cos x} \cdot dx$

$= 2\cos x - \sec x$

$= \boxed{2\sin x - \ln|\sec x + \tan x| + C}$

Q.2 $\int \tan x \tan 2x \tan 3x \, dx$

$\tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$

$\therefore \tan 3x - \tan 2x - \tan x = \tan 3x \tan 2x \tan x$

$\therefore \boxed{\frac{1}{3} \ln \sec 3x - \frac{1}{2} \ln \sec 2x - \ln \sec x + C}$

Q.3 $\int \frac{x^4 + 2x^3 + 1}{x^6} \cdot dx$

$= \int x^{-2} + 2x^{-3} + x^{-6}$

$= \boxed{-x^{-1} - 1x^{-2} - \frac{1}{5}x^{-5} + C}$

Q4 $\int \frac{x^4}{1+x^2} \cdot dx$

$= \int \frac{x^4-1}{1+x^2} + \int \frac{1}{1+x^2}$

$= \int x^2 - 1$

$= \frac{x^3}{3} - x + \tan^{-1}x + c$

Q.5 $\int \cos x^\circ \, dx$

$= \cos \frac{x\pi}{180} = \boxed{\frac{180}{\pi}\sin x^\circ} + c$

Q.6 $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} \cdot dx$

$= \int \frac{\sin^4 x + \cos^4 x - \cos^2 x \sin^2 x}{\sin^2 x \cos^2 x}$

$= \int \frac{1}{\sin^2 \cos^2 x} - \int 3 = \tan x - \cot x - 3x + c$

Q.7 $\int \frac{x}{x^2+2x+1} \cdot dx = \int \frac{1}{(x+1)} - \frac{1}{(x+1)^2}$

$= \frac{x}{(x+1)^2}$ $\quad \frac{1}{(x+1)^2} = \boxed{\ln(x+1) + \frac{1}{(x+1)} + c}$

Q.8 $\int \frac{dx}{x^6+x^4} = \int \frac{1}{x^4} - \frac{1}{x^2} + \frac{1}{x^2+1} = \boxed{\frac{-1}{3x^3} + \frac{1}{x} + \tan^{-1}x + c}$

Q.9 $\int 2^x \cdot e^x \cdot dx$

$= \int (2e)^x = \boxed{\dfrac{(2e)^x}{\ln 2e}} + C$

Q.10 $\int \left(\dfrac{\sin x}{\cos 3x} + \dfrac{\sin 3x}{\cos 9x} + \dfrac{\sin 9x}{\cos 27x}\right) \cdot dx$

$\int \frac{1}{2}\left(\tan 27x - \tan x\right)$.

$= \boxed{\dfrac{1}{2}\left(\dfrac{\ln \sec 27x}{27} - \ln \sec x\right)}$

Q.11 $\int \dfrac{dx}{1+\sin x}$

$= \dfrac{1-\sin x}{\cos^2 x} = \int \sec^2 x - \sec x \tan x$

$= \boxed{\tan x - \sec x + C}$

Q.12 $\int \dfrac{(\cos 8x - \cos 7x)}{(1+2\cos 5x)} \cdot dx$

$\boxed{-\dfrac{\sin 2x}{2} + \dfrac{\sin 3x}{3} +}$

$\dfrac{\sin 3x}{\sin x} = 1 + 2\cos 2x$

$= 2\sin\frac{x}{2}\sin\frac{5x}{2}$

$= \cos 3x - \cos 2x$

$= -\dfrac{\cancel{\sin\frac{15x}{2}}}{\cancel{\sin\frac{5x}{2}}} \times \dfrac{2\sin\frac{15x}{2}\sin\frac{x}{2}}{\sin\frac{15x}{2}} \cdot \sin\frac{5x}{2}$

Q13 $\int \frac{dx}{\cos(x+2)\cos(x+1)}$

$= \frac{1}{\sin 1}\left(\frac{\sin((x+2)-(x+1))}{\cos(x+2)\cos(x+1)}\right) = \frac{1}{\sin 1}\left(\tan(x+2) - \cot(x+1)\right)$

$= \boxed{\frac{1}{\sin 1}\left(\ln \sec(x+2) - \ln \sin(x+1)\right) + c}$

Q14 $\int \left(\frac{2x+3}{3x+2}\right) . dx$

$= \frac{2}{3}\left(\frac{3x+2}{3x+2}\right) + \frac{5}{3(3x+2)}$

$= \frac{2}{3}x + \frac{5}{9}\ln(3x+2) + c$

Q.15 $\int \frac{(x^2+\sin^2 x)\sec^2 x}{(1+x^2)} . dx = \int \frac{x^2\sec^2 x}{1+x^2} + \frac{\tan^2 x}{x^2+1}$

$= \int \sec^2 x - \frac{\sec^2 x}{x^2+1} + \frac{\tan^2 x}{x^2+1} = \int\left(\sec^2 x - \frac{1}{x^2+1}\right)$

$= \boxed{\tan x - \tan^{-1} x + c}$

Q.16 $\int \left(\frac{\cos 6x + 6\cos 4x + 15\cos 2x + 10}{10\cos^2 x + 5\cos x\cos 3x + \cos x\cos 5x}\right) . dx$

$= \int \left(\frac{\cos 6x + 6\cos 4x + 15\cos 2x + 10}{\frac{\cos 6x + \cos 4x}{2} + \frac{5}{2}(\cos 4x + \cos 2x) + 5\cos 2x + 5}\right) . dx$

$= \int 2 = \boxed{2x + c}$