

Ex $\int \frac{dx}{\sin^2 x \cos^2 x} \rightarrow \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}$
 $= \int \sec^2 x + \operatorname{cosec}^2 x = \tan x - \cot x + C$

$\int \frac{4}{\sin^2 2x} \cdot dx = 4 \int \operatorname{cosec}^2 2x \cdot dx = \frac{4 \cot 2x}{2} + C$
 $= 2 \cot 2x + C$

Question

Q.1 $\int \frac{\cos 2x}{\cos x} \cdot dx$

$= 2 \cos x - \sec x$

$= 2 \sin x - \ln |\sec x + \tan x| + C$

Q.2 $\int \tan x \tan 2x \tan 3x \cdot dx$

$\tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$

$\therefore \tan 3x - \tan 2x - \tan x = \tan 3x \tan 2x \tan x$

$\therefore \left[\frac{1}{3} \ln \sec 3x - \frac{1}{2} \ln \sec 2x - \ln \sec x + C \right]$

Q.3 $\int \frac{(x^4 + 2x^3 + 1)}{x^6} \cdot dx$

$= \int x^{-2} + 2x^{-3} + x^{-6}$

$= -x^{-1} - 1x^{-2} - \frac{1}{5}x^{-5} + C$

Q.4 $\int \frac{x^4}{1+x^2} dx$

$$= \int \frac{x^4-1}{1+x^2} + \frac{1}{1+x^2}$$

$$= \int x^2 - 1 + \frac{1}{1+x^2}$$

$$= \frac{x^3}{3} - x + \tan^{-1}x + c$$

Q.5 $\int \cos x^\circ dx$

$$= \cos \frac{x\pi}{180} = \frac{180 \sin x^\circ}{\pi} + c$$

Q.6 $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx$

$$= \int \frac{\sin^4 x + \cos^4 x - \cos^2 x \sin^2 x}{\sin^2 x \cos^2 x}$$

$$= \int \frac{1}{\sin^2 x \cos^2 x} - 3 = -\tan x - \cot x - 3x + c$$

Q.7 $\int \frac{x}{x^2+2x+1} dx$

$$= \int \frac{1}{(x+1)} - \frac{1}{(x+1)^2}$$

$$= \ln(x+1) + \frac{1}{(x+1)} + c$$

Q.8 $\int \frac{dx}{x^6+x^4}$

$$= \int \frac{1}{x^4} - \frac{1}{x^2} + \frac{1}{x^2+1}$$

$$= \frac{-1}{3x^3} + \frac{1}{x} + \tan^{-1}x + c$$

$$Q.9 \int 2^x e^x \cdot dx$$

$$= \int (2e)^x = \boxed{\frac{(2e)^x}{\ln 2e}} + C$$

$$Q.10 \int \left(\frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin 9x}{\cos 27x} \right) \cdot dx$$

$$\int \frac{1}{2} (\tan 27x - \tan x)$$

$$= \boxed{\frac{1}{2} \left(\frac{\ln \sec 27x}{27} - \frac{\ln \sec x}{1} \right)}$$

$$Q.11 \int \frac{dx}{1 + \sin x}$$

$$= \frac{1 - \sin x}{\cos 2x} = \int \sec^2 x - \sec x \tan x$$

$$= \boxed{\tan x - \sec x + C}$$

$$Q.12 \int \frac{(\cos 8x - \cos 7x)}{(1 + 2 \cos 5x)} \cdot dx$$

$$\frac{\sin 3x}{\sin x} = 1 + 2 \cos 2x$$

$$= 2 \sin x \cos \frac{5x}{2} \cdot \frac{\sin \frac{5x}{2}}{2}$$

$$= \cos 3x - \cos 5x$$

$$= \frac{-\sin 2x}{2} \cdot \frac{2 \sin 15x \sin x}{2} \cdot \frac{\sin 5x}{2}$$

$$\frac{\sin x}{2} \cdot \frac{\sin 15x}{2}$$

$$\frac{-\sin 2x}{2} + \frac{\sin 3x}{3}$$

$$Q13 \int \frac{du}{\cos(x+2)\cos(x+1)}$$

$$= \frac{1}{\sin 1} \left(\frac{\sin(x+2) - \sin(x+1)}{\cos(x+2)\cos(x+1)} \right) = \frac{1}{\sin 1} (\tan(x+2) - \cot(x+1))$$

$$= \frac{1}{\sin 1} (\ln \sec(x+2) - \ln \sin(x+1)) + c$$

$$Q14 \int \left(\frac{2x+3}{3x+2} \right) \cdot dx$$

$$= \frac{2}{3} \left(\frac{3x+2}{3x+2} \right) + \frac{5}{3} \frac{1}{3x+2}$$

$$= \frac{2}{3}x + \frac{5}{9} \ln(3x+2) + c$$

$$Q15 \int \frac{(x^2 + \sin^2 x) \sec^2 x}{(1+x^2)} \cdot dx = \int \left(\frac{x^2 \sec^2 x}{1+x^2} + \frac{\tan^2 x}{x^2+1} \right)$$

$$= \int \frac{\sec^2 x}{x^2+1} - \frac{\sec^2 x}{x^2+1} + \frac{\tan^2 x}{x^2+1} = \int \left(\sec^2 x - \frac{1}{x^2+1} \right)$$

$$= \boxed{\tan x - \tan^{-1} x + c}$$

$$Q16 \int \left(\frac{\cos 6x + 6 \cos 4x + 15 \cos 2x + 10}{10 \cos^2 x + 5 \cos x \cos 3x + \cos x \cos 5x} \right) \cdot dx$$

$$= \int \left(\frac{\cos 6x + 6 \cos 4x + 15 \cos 2x + 10}{\frac{\cos 6x + \cos 4x}{2} + 5(\cos 4x + \cos 2x) + 5 \cos 2x + 5} \right) \cdot dx$$

$$= \int 2 \cdot dx = \boxed{2x + c}$$