Q. $\int x\sqrt{\frac{2\sin(x^2-1)-\sin 2(x^2-1)}{2\sin(x^2-1)+\sin 2(x^2-1)}}\,dx$ [JEE-MAIN - 2019]

is equal to

(a) $\frac{1}{2}\log_e|\sec(x^2-1)|+c$ (b) $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c$

(c) $\log_e\left|\frac{1}{2}\sec^2(x^2-1)\right|+c$ (d) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

where c is constant of integration.

Solution Let we initally put $x^2-1=t$ to make problem a bit simpler

$\therefore x^2-1=t$

$2x\cdot dx = dt$ (diff. both sides)

we replace x^2-1 with t and $x\,dx$ with $\frac{dt}{2}$ in above problem

$\therefore \int \frac{1}{2}\sqrt{\frac{2\sin t-\sin 2t}{2\sin t+\sin 2t}}\,dt$ Now $\sin 2t = 2\sin t\cos t$

$= \int \frac{1}{2}\sqrt{\frac{2\sin t - 2\sin t\cos t}{2\sin t + 2\sin t\cos t}}\,dt = \int \frac{1}{2}\sqrt{\frac{1-\cos t}{1+\cos t}}\cdot dt$ (take $2\sin t$ common)

Now put $\cos t = 2\cos^2\frac{t}{2}-1$ (in denominator)

$\cos t = 1-2\sin^2\frac{t}{2}$ (in above part)

$= \int \frac{1}{2}\sqrt{\tan^2\frac{t}{2}}\,dt = \int\frac{1}{2}\left|\tan\frac{t}{2}\right|\cdot dt = \int\frac{1}{2}\left|\tan\frac{t}{2}\,dt\right| = \int\frac{1}{2}\tan\frac{t}{2}\,dt$

$= 2\times\frac{1}{2}\log_e\left|\sec\frac{t}{2}\right|+C$ We know $\int\tan x\,dx = \log_e|\sec x|+c$

$= \log_e\left(\left|\sec\frac{(x^2-1)}{2}\right|\right)+C$ $\therefore \int\tan\frac{x}{2}\,dx = 2\log_e\left|\sec\frac{x}{2}\right|+c$

$\therefore$ Option (b) is correct

Q. Evaluate $\int \frac{dx}{\sin x + \cos x}$ (JEE MAINS)

Solutions

we know $\sin x + \cos x = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x + \frac{\cos x}{\sqrt{2}}\right)$

$= \sqrt{2}\left(\sin x \cos 45 + \cos x \sin 45\right)$ $\left(\sin A \cos B + \cos A \sin B = \sin(A+B)\right)$

$= \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$

$= \int \frac{dx}{\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)} = \frac{1}{\sqrt{2}}\int \operatorname{cosec}\left(x+\frac{\pi}{4}\right).dx$

we known $\int \operatorname{cosec} x \, dx = \log_e \left|\tan\frac{x}{2}\right| + c$

$= \boxed{\frac{1}{\sqrt{2}} \log_e \left|\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)\right| + c}$