Indefinite Integration

Let $\frac{d}{dx}F(x) = f(x)$ $\frac{d}{dx}(F(x)+d) = f(x)$

$\therefore \int f(x).dx = F(x) + C$ [where C is called constant of integration]

→ Integration ⇒ Antiderivative, primitive

Properties of integration

① $\int a f(x).dx = a\int f(x).dx$ where a is constant

② $\int (k_1 f(x) \pm k_2 g(x)).dx = k_1\int f(x).dx \pm k_2\int g(x)$
k_1, k_2 are constant

Integration of some standard functions

1. $\int \frac{1}{\sqrt{a^2-x^2}}.dx = \sin^{-1}\left(\frac{x}{a}\right) + C = -\cos^{-1}\frac{x}{a} + C$

2. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C = -\frac{1}{a}\cot^{-1}\frac{x}{a} + C$

3. $\int \frac{1}{|x|\sqrt{x^2-a^2}}.dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C = -\frac{1}{a}\operatorname{cosec}^{-1}\frac{x}{a} + C$

4. $\int \cot x.dx = \int \frac{\cos x}{\sin x}.dx = \ln|\sin x| + C = -\ln|\operatorname{cosec} x| + C$

5. $\int \tan x.dx = -\int \frac{-\sin x}{\cos x}.dx = -\ln|\cos x| + C$
$= \ln|\sec x| + C$

6. $\int \sec x \, dx = \int \frac{\sec x(\sec x + \tan x)}{(\sec x + \tan x)} \, dx = \ln|\sec x + \tan x| + c$

$= \boxed{\ln\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + c}$

$= \boxed{-\ln|(\sec x - \tan x)| + c}$

$= \cancel{-\ln(\cot x + \operatorname{cosec} x)| + c}$

7 $\int \operatorname{cosec} x \, dx = \int \frac{\operatorname{cosec} x(\cot x - \operatorname{cosec} x)}{(\cot x - \operatorname{cosec} x)} \, dx = \ln|(\cot x - \operatorname{cosec} x)| + c$

$= \ln\left|\tan\frac{x}{2}\right| + c$

$= -\ln|(\cot x + \operatorname{cosec} x)| + c$

8 $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|x + \sqrt{x^2+a^2}\right| + c$

9 $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x + \sqrt{x^2-a^2}\right| + c$

10 $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c$

11 $\int \frac{du}{a^2-u^2} = \frac{1}{2a} \ln\left|\frac{a+u}{a-u}\right| + c$

12 $\int \sqrt{u^2+a^2} \, du = \frac{u}{2}\sqrt{u^2+a^2} + \frac{a^2}{2} \ln\left|u + \sqrt{u^2+a^2}\right| + c$

13 $\int \sqrt{u^2-a^2} \, du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2} \ln\left|u + \sqrt{u^2-a^2}\right| + c$

14 $\int \sqrt{a^2-x^2} \, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1}\frac{x}{a} + c$

15 $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)+c$

$$= \frac{e^{ax}}{\sqrt{a^2+b^2}}\sin\left(bx-\tan^{-1}\frac{b}{a}\right)+c$$

16 $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)+c$

$$= \frac{e^{ax}}{\sqrt{a^2+b^2}}\cos\left(bx-\tan^{-1}\frac{b}{a}\right)+c$$

→ Let $\int f(x)\cdot dx = F(x)+c$ then

$$\boxed{\int f(ax+b)\,dx = \frac{1}{a}F(ax+b)+c}$$

a, b are constant

Eg– $\int \frac{dx}{\sqrt{a^2-(cx+d)^2}} = \boxed{\frac{1}{c}\sin^{-1}\frac{(cx+d)}{a}+\lambda}$

Eg– ① $\int \tan^2 x\,dx = \int(\sec^2 x-1)\,dx = \tan x - x + c$

② $\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx$

$$= \frac{1}{2}\left(x-\frac{\sin 2x}{2}\right)+c$$

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(3) $\int \frac{dx}{\sqrt{x+1}+\sqrt{x}} = \int (\sqrt{x+1} - \sqrt{x}) \cdot dx = \boxed{\frac{(x+1)^{3/2}}{3/2} - \frac{x^{3/2}}{3/2} + c}$

for integration Use
- Standard formulas
- Changing to easy function
- Rationalisation
- for sinx ~~cos~~ give $1 = \sin^2 x + \cos^2 x$ use this

(4) $\int \sqrt{1 - \sin 2x} \cdot dx$

$\int \sqrt{(\sin^2 x + \cos^2 x - 2 \sin x \cos x)} \cdot dx = \int (\sin x - \cos x) \cdot dx$

$\int (\sin x - \cos x) \cdot dx = \boxed{-\cos x - \sin x + c}$ $\quad$ $-\int (\sin x - \cos x) \cdot dx = -(-\sin x - \cos x) + c$

* for this chapter (for general questions) → we don't use modulus here

$\int \sqrt{x^2} \cdot dx = \int x \cdot dx = \boxed{\frac{x^2}{2} + c}$

→ $\int \frac{1}{x} \cdot dx = \ln|x| \Rightarrow$ general given $\boxed{\ln x + c}$

17. $\int \sin x \cdot dx = -\cos x + c$

18. $\int \sec^2 x \cdot dx = \tan x + c$

19. $\int \csc^2 x \cdot dx = -\cot x + c$