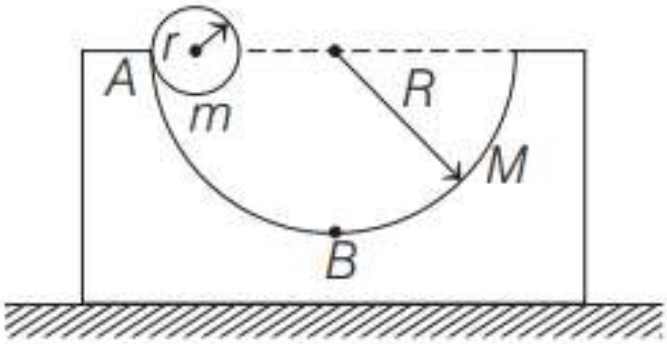
A block of mass M with a semicircular track of radius R, rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest at the top point A (see fig.). The cylinder slips on the semicircular frictionless track.

(a) How far has the block moved when the cylinder reaches the bottom (point B) of the track ?

(b) How fast is the block moving when the cylinder reaches the bottom of the track? **(1983, 7M)**



(a) The centre of mass of $M + m$ in this case will not move in horizontal direction. Let M moves towards left by a distance x then m will move towards right by a distance $R - r - x$ (with respect to ground). For centre of mass not to move along horizontal we should have

$$Mx = m(R - r - x), \quad x = \frac{m(R - r)}{M + m}$$

(b) Let v_1 be the speed of m towards right and v_2 the speed of M towards left. From conservation of linear momentum.

$$mv_1 = Mv_2 \quad \ldots(\text{i})$$

From conservation of mechanical energy

$$mg(R - r) = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2 \quad \ldots(\text{ii})$$

Solving these two equations, we get

$$v_2 = m\sqrt{\frac{2g(R - r)}{M(M + m)}}$$