

Question 1.

Find the equation of the circle touching the line $2x + 3y + 1 = 0$ at the point $(1, -1)$ and is orthogonal to the circle which has the line segment having end points $(0, -1)$ and $(-2, 3)$ as the diameter.

Solution:

The equation of the circle touching the line $2x + 3y + 1 = 0$ at the point $(1, -1)$ is

$$(x-1)^2 + (y+1)^2 + \lambda(2x + 3y + 1) = 0$$

$$\Rightarrow x^2 + y^2 + 2x(\lambda-1) + y(3\lambda+2) + (\lambda+2) = 0 \dots (1)$$

it is orthogonal to the circle which have end point of diameter $(0, -1)$ and $(-2, 3)$.

It gives

$$x(x+2) + (y+1)(y-3) = 0$$

$$\Rightarrow x^2 + y^2 + 2x - 2y - 3 = 0$$

$$\text{so, } 2(\lambda - 2) / 2 \cdot 1 + 2(3\lambda + 2) / 2 \cdot (-1) = \lambda + 2 - 3$$

This gives $\lambda = -3/2$

hence, from equation (1), the equation of circle is $2x^2 + 2y^2 - 10x - 5y + 1 = 0$.