Chapter 12 Linear Programming

NCERT Problems

Q1) An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps, D, E and F whose requirements are 4500L, 3000L and 3500L respectively. The distances (in km) between the depots and the petrol pumps is given in the following table:

Distance (in Kms)		
From/To	**A**	**B**
D	**7**	**3**
E	**6**	**4**
F	**3**	**2**

Assuming that the transportation cost of 10 litres of oil is Re 1 per km, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost?

Ans 1) Let x and y litres of oil be supplied from A to the petrol pumps, D and E. Then, (7000 − x − y) will be supplied from A to petrol pump F. The requirement at petrol pump D is 4500 L. Since x L are transported from depot A, the remaining (4500 −x) L will be transported from petrol pump B. Similarly, (3000 − y) L and 3500 − (7000 − x − y) = (x + y − 3500) L will be transported from depot B to petrol pump E and F respectively.

Now,

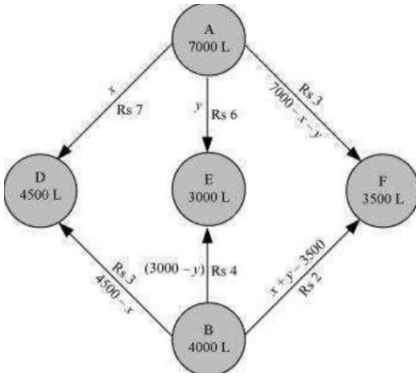
$x \geq 0$, $y \geq 0$, $(7000-x-y) \geq 0$,

$\rightarrow$ $x \geq 0$, $y \geq 0$, $x+y \leq 7000$

$4500-x \geq 0$, $3000-y \geq 0$, $x+y-3500 \geq 0$,

$\rightarrow$ $x \leq 4500$, $y \leq 3000$, $x+y \geq 3500$

Diagramatically,



Now, Cost of transporting 10 L of petrol = Re 1

Cost of transporting 1 L of petrol = Rs $\frac{1}{10}$

Therefore, total transportation cost is given by,

$Z=\frac{7}{10}\times x+\frac{6}{10}\times y+\frac{3}{10}(7000-x-y)+\frac{3}{10}(4500-x)+\frac{4}{10}(3000-y)+\frac{2}{10}(x+y-3500)$

$\rightarrow$ Z=0.3x+0.1y+3950…….(1)

Now according to problem we have to minimize Z

Subject to constraints-

x≤ 4500……..(2)

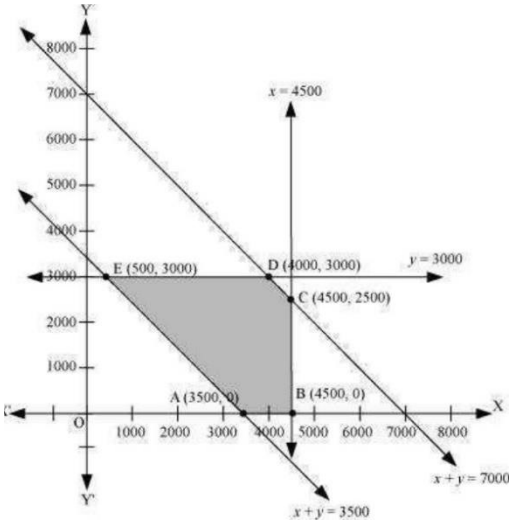
y≤ 3000……(3)

x+y≥3500……(4)

x+y ≤ 7000……(5)

x, $y \geq 0$……(6)

The feasible region determined by the constraints is as follows.



The corner points of the feasible region are A (3500, 0), B (4500, 0), C (4500, 2500), D (4000, 3000), and E (500, 3000). The values of Z at these corner points are as follows.

Corner Point	Z=0.3x+0.1y+3950
A (3500, 0)	5000
B (4500, 0)	5300
C (4500, 2500)	5550
D (4000, 3000)	5450
E (500, 3000)	4400 (Minimum)

The minimum value of Z is 4400 at (500, 3000).

Thus, the oil supplied from depot A is 500 L, 3000 L, and 3500 L and from depot B is 4000 L, 0 L, and 0 L to petrol pumps D, E, and F respectively.

The minimum transportation cost is Rs 4400.

Q2) A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin content of one kg food is given below:

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

One kg of food X costs Rs 16 and one kg of food Y costs Rs 20. Find the least cost of the mixture which will produce the required diet?

Ans 2) Let the mixture contain x kg of food X and y kg of food Y. The mathematical formulation of the given problem is as follows.

Minimize z = 16x + 20y ... (1) subject to the constraints,

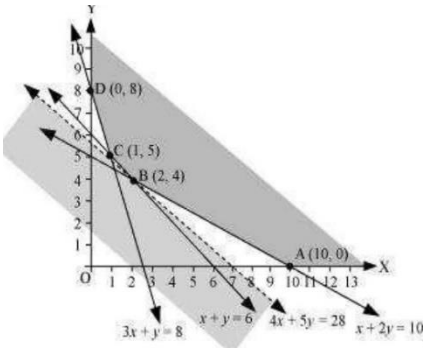
$x+2y \geq 10$.....(2)

$x+y \geq 6$.....(3)

$3x+y \geq 8$.....(4)

$x, y \geq 0$.....(5)

The feasible region determined by the system of constraints is as follows.



The corner points of the feasible region are A (10, 0), B (2, 4), C (1, 5), and D (0, 8). The values of z at these corner points are as follows.

Corner Point	z = 16x + 20y
A(10,0)	160
B(2,4)	112 (Minimum)
C(1,5)	116
D(0,8)	160

As the feasible region is unbounded, therefore, 112 may or may not be the minimum value of z.
For this, we draw a graph of the inequality, $16x + 20y < 112$ or $4x + 5y < 28$, and check whether the resulting half plane has points in common with the feasible region or not.
It can be seen that the feasible region has no common point with $4x + 5y < 28$
Therefore, the minimum value of z is 112 at (2, 4).
Thus, the mixture should contain 2 kg of food X and 4 kg of food Y. The minimum cost of the mixture is Rs 112.