Lecture 4

Definite Integral

Exemplar Problem

Example 14 Find the area of the region

$$\{(x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$$

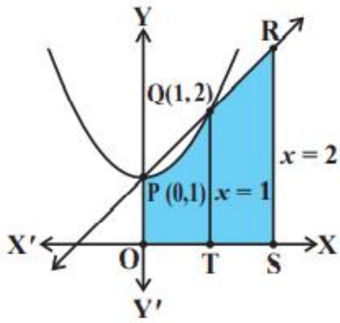

Fig 8.24

Solution Let us first sketch the region whose area is to be found out. This region is the intersection of the following regions.

$$A_1 = \{(x, y) : 0 \leq y \leq x^2 + 1\},$$

$$A_2 = \{(x, y) : 0 \leq y \leq x + 1\}$$

and

$$A_3 = \{(x, y) : 0 \leq x \leq 2\}$$

The points of intersection of $y = x^2 + 1$ and $y = x + 1$ are points P(0, 1) and Q(1, 2). From the Fig 8.24, the required region is the shaded region OPQRSTO whose area

= area of the region OTQPO + area of the region TSRQT

$$= \int_0^1 (x^2 + 1)\, dx + \int_1^2 (x + 1)\, dx \qquad \text{(Why?)}$$

$$= \left[\left(\frac{x^3}{3} + x\right)\right]_0^1 + \left[\left(\frac{x^2}{2} + x\right)\right]_1^2$$

$$= \left[\left(\frac{1}{3} + 1\right) - 0\right] + \left[(2 + 2) - \left(\frac{1}{2} + 1\right)\right] = \frac{23}{6}$$