Lecture 4

Definite Integral

Exemplar Problem

Example 4 Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$, and the circle $x^2 + y^2 = 32$.

Solution The given equations are

$$y = x \qquad \text{... (1)}$$

and $$x^2 + y^2 = 32 \qquad \text{... (2)}$$

Solving (1) and (2), we find that the line and the circle meet at B(4, 4) in the first quadrant (Fig 8.11). Draw perpendicular BM to the x-axis.

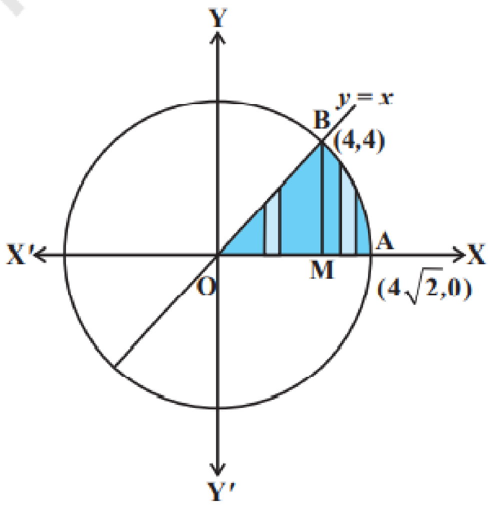

Fig 8.11

Therefore, the required area = area of the region OBMO + area of the region BMAB.

Now, the area of the region OBMO

$$= \int_0^4 y\,dx = \int_0^4 x\,dx \qquad \text{... (3)}$$

$$= \frac{1}{2}\left[x^2\right]_0^4 = 8$$

Again, the area of the region BMAB

$$= \int_4^{4\sqrt{2}} y\,dx = \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\,dx$$

$$= \left[\frac{1}{2}x\sqrt{32 - x^2} + \frac{1}{2} \times 32 \times \sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$

$$= \left(\frac{1}{2}4\sqrt{2} \times 0 + \frac{1}{2} \times 32 \times \sin^{-1}1\right) - \left(\frac{4}{2}\sqrt{32 - 16} + \frac{1}{2} \times 32 \times \sin^{-1}\frac{1}{\sqrt{2}}\right)$$

$$= 8\pi - (8 + 4\pi) = 4\pi - 8 \qquad \text{... (4)}$$

Adding (3) and (4), we get, the required area = 4π.