Lecture 4

Definite Integral

Exemplar Problem

Example 8 In Fig 8.17, AOBA is the part of the ellipse $9x^2 + y^2 = 36$ in the first quadrant such that OA = 2 and OB = 6. Find the area between the arc AB and the chord AB.

Solution Given equation of the ellipse $9x^2 + y^2 = 36$ can be expressed as $\dfrac{x^2}{4} + \dfrac{y^2}{36} = 1$ or

$\dfrac{x^2}{2^2} + \dfrac{y^2}{6^2} = 1$ and hence, its shape is as given in Fig 8.17.

Accordingly, the equation of the chord AB is

$$y - 0 = \dfrac{6-0}{0-2}(x-2)$$

or $\qquad\qquad y = -3(x-2)$

or $\qquad\qquad y = -3x + 6$

Area of the shaded region as shown in the Fig 8.17.

$$= 3\int_0^2 \sqrt{4-x^2}\,dx - \int_0^2 (6-3x)\,dx \qquad \text{(Why?)}$$

$$= 3\left[\dfrac{x}{2}\sqrt{4-x^2} + \dfrac{4}{2}\sin^{-1}\dfrac{x}{2}\right]_0^2 - \left[6x - \dfrac{3x^2}{2}\right]_0^2$$

$$= 3\left[\dfrac{2}{2}\times 0 + 2\sin^{-1}(1)\right] - \left[12 - \dfrac{12}{2}\right] = 3\times 2\times\dfrac{\pi}{2} - 6 = 3\pi - 6$$
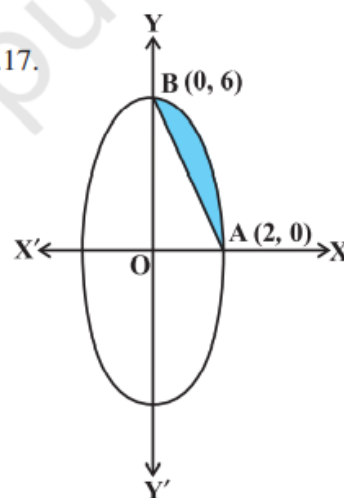
Fig 8.17