Lecture 4

Definite Integral

Exemplar Problem

Example 9 Using integration find the area of region bounded by the triangle whose vertices are (1, 0), (2, 2) and (3, 1).

Solution Let A (1, 0), B (2, 2) and C (3, 1) be the vertices of a triangle ABC (Fig 8.18).

Area of ΔABC

= Area of ΔABD + Area of trapezium BDEC − Area of ΔAEC
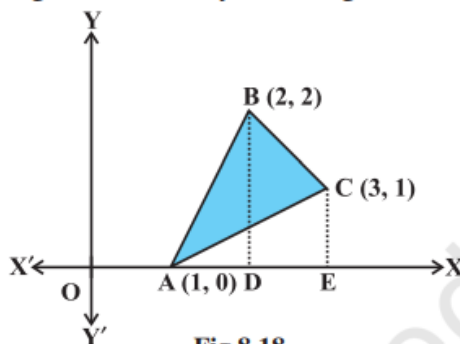
Now equation of the sides AB, BC and CA are given by

Fig 8.18

$$y = 2(x-1), y = 4 - x, y = \frac{1}{2}(x-1), \text{ respectively.}$$

Hence, area of Δ ABC $= \int_1^2 2(x-1)\,dx + \int_2^3 (4-x)\,dx - \int_1^3 \frac{x-1}{2}\,dx$

$$= 2\left[\frac{x^2}{2} - x\right]_1^2 + \left[4x - \frac{x^2}{2}\right]_2^3 - \frac{1}{2}\left[\frac{x^2}{2} - x\right]_1^3$$

$$= 2\left[\left(\frac{2^2}{2} - 2\right) - \left(\frac{1}{2} - 1\right)\right] + \left[\left(4\times3 - \frac{3^2}{2}\right) - \left(4\times2 - \frac{2^2}{2}\right)\right] - \frac{1}{2}\left[\left(\frac{3^2}{2} - 3\right) - \left(\frac{1}{2} - 1\right)\right]$$

$$= \frac{3}{2}$$