Lecture 4

Definite Integral

Related Problem with Solution

Que 1:

Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.

Solution:

Area is bounded by the circle $4x^2 + 4y^2 = 9$ and interior of the parabola $x^2 = 4y$.

Putting $x^2 = 4y$ in $x^2 + y^2 = \frac{9}{4}$

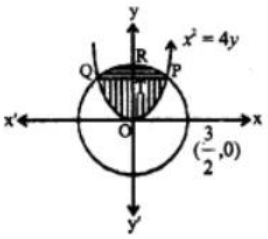
We get $4y + y^2 = \frac{9}{4}$

or $4y^2 + 16y - 9 = 0$,

$(2y+9)(2y-1) = 0$

$\Rightarrow y = \frac{1}{2}, -\frac{9}{2}$

$\Rightarrow y \neq -\frac{9}{2} \quad \therefore \quad y = \frac{1}{2}$



Radius of the circle $= \frac{3}{2}$,

Area of the region required=Area of the region OPRQ.

= {(area of the region OPQ) + (area of the region PQR)]

= 2 × area of the region OPT + 2 × area of the region TPR

$$= 2 \cdot 2\int_0^{\frac{1}{2}} \sqrt{y}\,dy + 2\int_{1/2}^{3/2} \sqrt{\frac{9-4y^2}{4}}\,dy$$

$$= 4\int_0^{1/2} \sqrt{y}\,dy + 2\int_{1/2}^{3/2} \sqrt{\frac{9}{4} - y^2}\,dy$$

$$= 4 \times \frac{2}{3}\left[y^{\frac{3}{2}}\right]_0^{1/2} + 2\left[\frac{y}{2}\sqrt{\frac{9}{4} - y^2} + \frac{9}{8}\sin^{-1}\left(\frac{y}{3/2}\right)\right]_{1/2}^{3/2}$$

$$= \frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\left(1 \cdot \frac{2\sqrt{2}}{3} - 0\right) = \frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$$

Que 2:

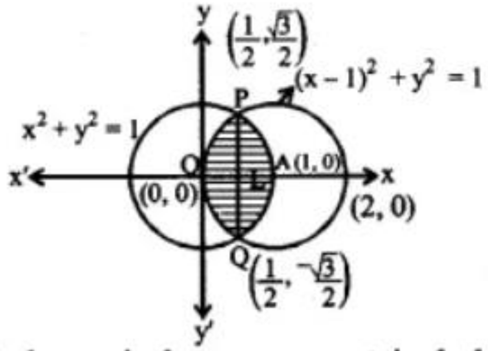
Find the area bounded by curves $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$.

Solution:

Given circles are $x^2 + y^2 = 1$ ...(i)

and $(x - 1)^2 + y^2 = 1$ ...(ii)

Centre of (i) is O (0,0) and radius = 1



Both these circle are symmetrical about x-axis

solving (i) and (ii) we get, $-2x + 1 = 0 \Rightarrow x = \frac{1}{2}$

then $y^2 = 1 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} \Rightarrow y = \frac{\sqrt{3}}{2}$

$\therefore$ The points of intersection are

$P\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $Q\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

It is clear from the figure that the shaded portion in region whose area is required.

$\therefore$ Req. area = area OQAPO = 2 × area of the region OLAP = 2 × (area of the region OLPO + area of LAPL)

$$= 2\left[\int_0^{1/2} \sqrt{1-(x-1)^2}\,dx + \int_{1/2}^{1} \sqrt{1-x^2}\,dx\right]$$

$$= 2\left[\frac{(x-1)\sqrt{1-(x-1)^2}}{2} + \frac{1}{2}\sin^{-1}(x-1)\right]_0^{1/2} + 2\left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1}x\right]_{1/2}^{1}$$

$$= \frac{\sqrt{3}}{4} - \frac{\pi}{6} - \left(-\frac{\pi}{2}\right) + \frac{\pi}{2} - \frac{\sqrt{3}}{4} - \frac{\pi}{6} = \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$$

sq. units.