(ii) The object distance $u = –5$ cm. Then from Eq. (9.7),

$$\frac{1}{v} \quad \frac{1}{5} \quad \frac{1}{7.5}$$

or $$v = \frac{5 \quad 7.5}{7.5 - 5} \quad 15 \text{ cm}$$

This image is formed at 15 cm behind the mirror. It is a virtual image.

Magnification $m = -\frac{v}{u} \quad -\frac{15}{(\ 5)} \quad 3$

The image is magnified, virtual and erect.

EXAMPLE 9.3

Example 9.4 Suppose while sitting in a parked car, you notice a jogger approaching towards you in the side view mirror of $R = 2$ m. If the jogger is running at a speed of 5 m s^{-1}, how fast the image of the jogger appear to move when the jogger is (a) 39 m, (b) 29 m, (c) 19 m, and (d) 9 m away.

Solution

From the mirror equation, Eq. (9.7), we get

$$v \quad \frac{fu}{u \quad f}$$

For convex mirror, since $R = 2$ m, $f = 1$ m. Then

for $u = –39$ m, $v \quad \frac{(\ 39) \quad 1}{39 \quad 1} \quad \frac{39}{40}$ m

Since the jogger moves at a constant speed of 5 m s^{-1}, after 1 s the position of the image v (for $u = –39 + 5 = –34$) is (34/35)m.

The shift in the position of image in 1 s is

$$\frac{39}{40} \quad \frac{34}{35} \quad \frac{1365 \quad 1360}{1400} \quad \frac{5}{1400} \quad \frac{1}{280} \text{ m}$$

Therefore, the average speed of the image when the jogger is between 39 m and 34 m from the mirror, is (1/280) m s^{-1}

Similarly, it can be seen that for $u = –29$ m, $–19$ m and $–9$ m, the speed with which the image appears to move is

$$\frac{1}{150} \text{ m s}^{-1}, \frac{1}{60} \text{ m s}^{-1} \text{ and } \frac{1}{10} \text{ m s}^{-1}, \text{ respectively.}$$

Although the jogger has been moving with a constant speed, the speed of his/her image appears to increase substantially as he/she moves closer to the mirror. This phenomenon can be noticed by any person sitting in a stationary car or a bus. In case of moving vehicles, a similar phenomenon could be observed if the vehicle in the rear is moving closer with a constant speed.

EXAMPLE 9.4

9.3 REFRACTION

When a beam of light encounters another transparent medium, a part of light gets reflected back into the first medium while the rest enters the other. A ray of light represents a beam. The direction of propagation of an obliquely incident ray of light that enters the other medium, changes at the interface of the two media. This phenomenon is called *refraction of light*. Snell experimentally obtained the following laws of refraction:

(i) The incident ray, the refracted ray and the normal to the interface at the point of incidence, all lie in the same plane.

(ii) The ratio of the sine of the angle of incidence to the sine of angle of refraction is constant. Remember that the angles of incidence (i) and refraction (r) are the angles that the incident and its refracted ray make with the normal, respectively. We have

$$\frac{\sin i}{\sin r} \quad n_{21} \qquad (9.10)$$

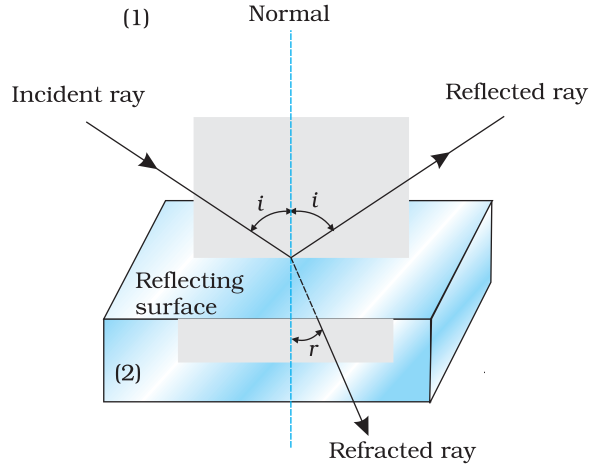


FIGURE 9.8 Refraction and reflection of light.

where n_{21} is a constant, called the *refractive index* of the second medium with respect to the first medium. Equation (9.10) is the well-known Snell's law of refraction. We note that n_{21} is a characteristic of the pair of media (and also depends on the wavelength of light), but is independent of the angle of incidence.

From Eq. (9.10), if $n_{21} > 1$, $r < i$, i.e., the refracted ray bends towards the normal. In such a case medium 2 is said to be *optically denser* (or *denser*, in short) than medium 1. On the other hand, if $n_{21} < 1$, $r > i$, the refracted ray bends away from the normal. This is the case when incident ray in a denser medium refracts into a rarer medium.

Note: *Optical density should not be confused with mass density, which is mass per unit volume. It is possible that mass density of an optically denser medium may be less than that of an optically rarer medium (optical density is the ratio of the speed of light in two media). For example, turpentine and water. Mass density of turpentine is less than that of water but its optical density is higher.*

If n_{21} is the refractive index of medium 2 with respect to medium 1 and n_{12} the refractive index of medium 1 with respect to medium 2, then it should be clear that

$$n_{12} \quad \frac{1}{n_{21}} \qquad (9.11)$$

It also follows that if n_{32} is the refractive index of medium 3 with respect to medium 2 then $n_{32} = n_{31} \times n_{12}$, where n_{31} is the refractive index of medium 3 with respect to medium 1.

Some elementary results based on the laws of refraction follow immediately. For a rectangular slab, refraction takes place at two interfaces (air-glass and glass-air). It is easily seen from Fig. 9.9 that $r_2 = i_1$, i.e., the emergent ray is parallel to the incident ray—there is no

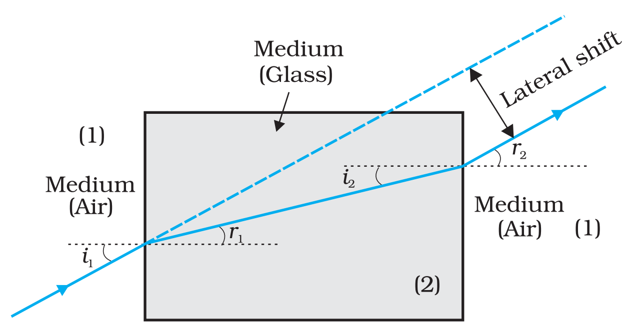


FIGURE 9.9 Lateral shift of a ray refracted through a parallel-sided slab.

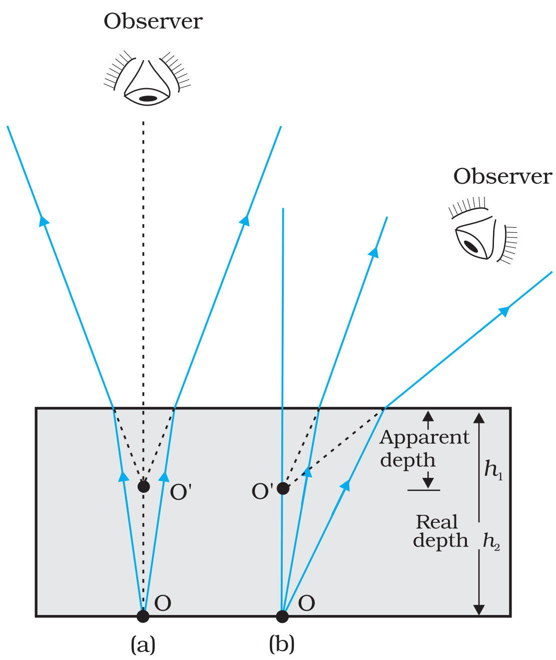


FIGURE 9.10 Apparent depth for (a) normal, and (b) oblique viewing.

deviation, but it does suffer lateral displacement/shift with respect to the incident ray. Another familiar observation is that the bottom of a tank filled with water appears to be raised (Fig. 9.10). For viewing near the normal direction, it can be shown that the apparent depth, (h_1) is real depth (h_2) divided by the refractive index of the medium (water).

The refraction of light through the atmosphere is responsible for many interesting phenomena. For example, the sun is visible a little before the actual sunrise and until a little after the actual sunset due to refraction of light through the atmosphere (Fig. 9.11). By actual sunrise we mean the actual crossing of the horizon by the sun. Figure 9.11 shows the actual and apparent positions of the sun with respect to the horizon. The figure is highly exaggerated to show the effect. The refractive index of air with respect to vacuum is 1.00029. Due to this, the apparent shift in the direction of the sun is by about half a degree and the corresponding time difference between actual sunset and apparent sunset is about 2 minutes (see Example 9.5). The apparent flattening (oval shape) of the sun at sunset and sunrise is also due to the same phenomenon.

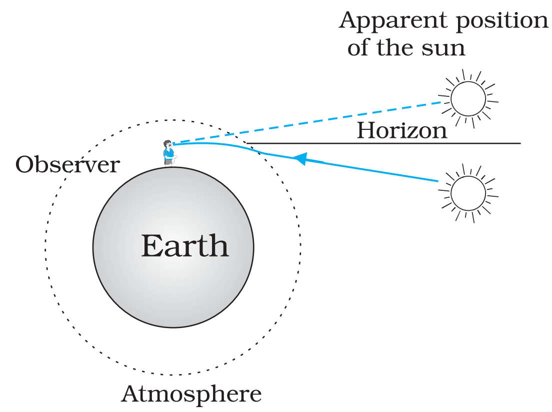


FIGURE 9.11 Advance sunrise and delayed sunset due to atmospheric refraction.

EXAMPLE 9.5

Example 9.5 The earth takes 24 h to rotate once about its axis. How much time does the sun take to shift by 1º when viewed from the earth?

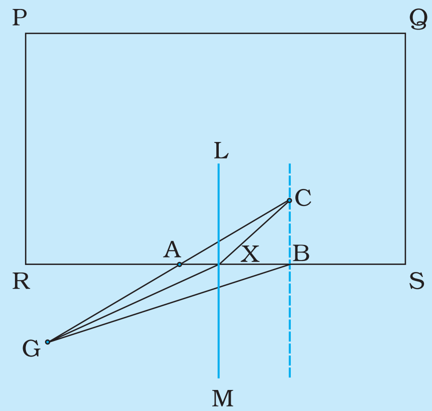
Solution

Time taken for 360° shift = 24 h

Time taken for 1° shift = 24/360 h = 4 min.

The drowning child, lifeguard and Snell's law

Consider a rectangular swimming pool PQSR; see figure here. A lifeguard sitting at G outside the pool notices a child drowning at a point C. The guard wants to reach the child in the shortest possible time. Let SR be the side of the pool between G and C. Should he/she take a straight line path GAC between G and C or GBC in which the path BC in water would be the shortest, or some other path GXC? The guard knows that his/her running speed v_1 on ground is higher than his/her swimming speed v_2.



Suppose the guard enters water at X. Let GX $= l_1$ and XC $= l_2$. Then the time taken to reach from G to C would be

$$t \quad \frac{l_1}{v_1} \quad \frac{l_2}{v_2}$$

To make this time minimum, one has to differentiate it (with respect to the coordinate of X) and find the point X when t is a minimum. On doing all this algebra (which we skip here), we find that the guard should enter water at a point where Snell's law is satisfied. To understand this, draw a perpendicular LM to side SR at X. Let $\angle$GXM = i and $\angle$CXL = r. Then it can be seen that t is minimum when

$$\frac{\sin i}{\sin r} \quad \frac{v_1}{v_2}$$

In the case of light v_1/v_2, the ratio of the velocity of light in vacuum to that in the medium, is the refractive index n of the medium.

In short, whether it is a wave or a particle or a human being, whenever two mediums and two velocities are involved, one must follow Snell's law if one wants to take the shortest time.

9.4 Total Internal Reflection

When light travels from an optically denser medium to a rarer medium at the interface, it is partly reflected back into the same medium and partly refracted to the second medium. This reflection is called the *internal reflection.*

When a ray of light enters from a denser medium to a rarer medium, it bends away from the normal, for example, the ray AO$_1$ B in Fig. 9.12. The incident ray AO$_1$ is partially reflected (O$_1$C) and partially transmitted (O$_1$B) or refracted, the angle of refraction (r) being larger than the angle of incidence (i). As the angle of incidence increases, so does the angle of refraction, till for the ray AO$_3$, the angle of refraction is $\pi/2$. The refracted ray is bent so much away from the normal that it grazes the surface at the interface between the two media. This is shown by the ray AO$_3$ D in Fig. 9.12. If the angle of incidence is increased still further (e.g., the ray AO$_4$), refraction is not possible, and the incident ray is totally reflected.