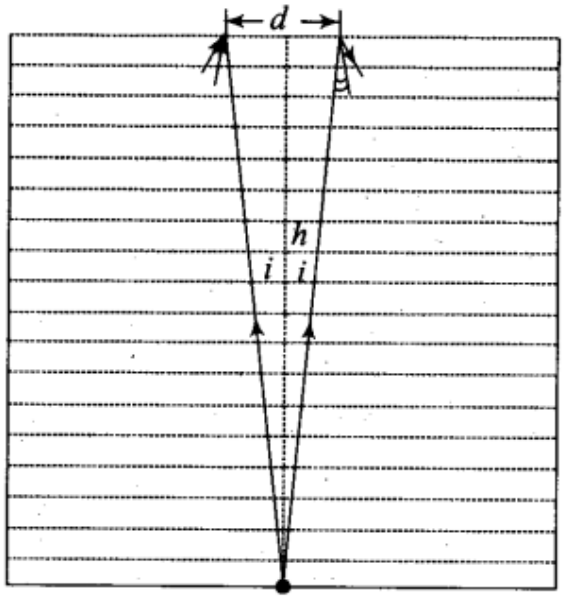
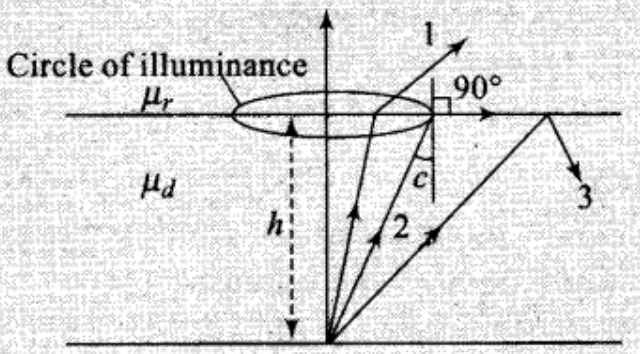
Question 5. A jar of height h is filled with a transparent liquid of refraction index μ (figure).At the center of the jar on the bottom surface is a dot.Find the minimum diameter of a disc,such that when placed on the top surface symmetrically about the center,the dot is invisible.



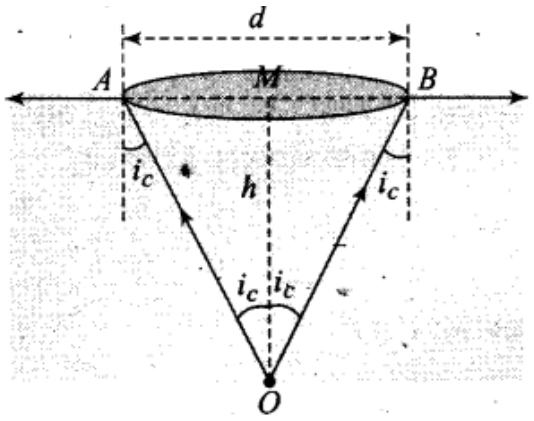
Solution:

Key concept:



In the figure, ray 1 strikes the surface at an angle less than critical angle c and gets refracted in rarer medium. Ray 2 strikes the surface at critical angle and grazes the interface. Ray 3 strikes the surface making an angle greater than the critical angle and gets internally reflected. The locus of points where ray strikes at critical angle is a circle, called ***circle of illuminance*** (C.O.I). All light rays striking inside the circle of illuminance get refracted in the rarer medium. If an observer is in the rarer medium, he/she will see light coming out only from within the circle of illuminance. If a circular opaque plate covers the circle of illuminance, no light will get refracted in the rarer medium and then the object cannot be seen from the rarer medium.

In figure, O is a small dot at the bottom of the jar. The ray from the dot emerges out of a circular patch of water surface of diameter AB till the angle of incidence for the rays OA and OB exceeds the critical angle (i_C).



Rays of light incident at an angle greater than i_C, are totally reflected within water and consequently cannot emerge out of the water surface.

As $\sin i_C = \frac{1}{\mu} \Rightarrow \tan i_C = \frac{1}{\sqrt{\mu^2 - 1}}$,

Now, $\frac{d/2}{h} = \tan i_c$

$\Rightarrow \quad \frac{d}{2} = h \tan i_c = h \frac{1}{\sqrt{\mu^2 - 1}}$

$\Rightarrow \quad d = \frac{2h}{\sqrt{\mu^2 - 1}}$

This is the required expression of d.