Sol. (c) We have,

$$\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$$

$\Rightarrow \qquad 1(2a^2 + 4) - 2(-4a - 20) + 0 = 86 \quad$ [Expanding along C_1]

$\Rightarrow \qquad a^2 + 4a - 21 = 0$

$\Rightarrow \qquad (a + 7)(a - 3) = 0$

$\Rightarrow \qquad a = -7$ and 3

$\therefore$ Required sum $= -7 + 3 = -4$

Sol. (c) We have $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

$\Rightarrow \qquad 2x^2 - 40 = 18 + 14$

$\Rightarrow \qquad 2x^2 = 72$

$\Rightarrow \qquad x^2 = 36$

$\therefore \qquad x = \pm 6$

Solution Applying $R_1 \rightarrow (R_1 + R_2 + R_3)$, we get

$$\begin{vmatrix} x+4 & x+4 & x+4 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix}.$$

Taking $(x + 4)$ common from R_1, we get

$$\Delta = (x+4) \begin{vmatrix} 1 & 1 & 1 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$, $C_3 \rightarrow C_3 - C_1$, we get

$$\Delta = (x+4) \begin{vmatrix} 1 & 0 & 0 \\ 1 & x-1 & 0 \\ 3 & -1 & x-3 \end{vmatrix}.$$

Expanding along R_1,

$\Delta = (x + 4) [(x - 1)(x - 3) - 0]$. Thus, $\Delta = 0$ implies

$$x = -4, 1, 3$$

Sol.
$$\begin{vmatrix} 1 & \cos C & \cos B \\ \cos C & 1 & \cos A \\ \cos B & \cos A & 1 \end{vmatrix}$$

$= 1(1 - \cos^2 A) - \cos C (\cos C - \cos A \cdot \cos B) + \cos B (\cos C \cdot \cos A - \cos B)$

$= \sin^2 A - \cos^2 C + \cos A \cdot \cos B \cdot \cos C + \cos A \cdot \cos B \cdot \cos C - \cos^2 B$

$= \sin^2 A - \cos^2 B + 2 \cos A \cdot \cos B \cdot \cos C - \cos^2 C$

$= -\cos (A + B) \cdot \cos (A - B) + 2 \cos A \cdot \cos B \cdot \cos C - \cos^2 C$

$$[\because \cos^2 B - \sin^2 A = \cos (A + B) \cdot \cos (A - B)]$$

$= -\cos (-C) \cdot \cos (A - B) + \cos C (2 \cos A \cdot \cos B - \cos C)$

$= -\cos C (\cos A \cdot \cos B + \sin A \cdot \sin B - 2 \cos A \cdot \cos B + \cos C)$

$= \cos C (\cos A \cdot \cos B - \sin A \cdot \sin B - \cos C)$

$= \cos C [\cos (A + B) - \cos C]$

$= \cos C (\cos C - \cos C) \qquad (\text{As } \cos C = \cos (A + B))$

$= 0$

Sol. We have,

$$\begin{vmatrix} 1 & 1 & \sin 3\theta \\ -4 & 3 & \cos 2\theta \\ 7 & -7 & -2 \end{vmatrix} = 0$$

Expanding along C_3, we get

$$\sin 3\theta \times (28 - 21) - \cos 2\theta \times (-7 - 7) - 2(3 + 4) = 0$$

$\Rightarrow \quad 7 \sin 3\theta + 14 \cos 2\theta - 14 = 0$

$\Rightarrow \quad \sin 3\theta + 2 \cos 2\theta - 2 = 0$

$\Rightarrow \quad (3 \sin \theta - 4 \sin^3 \theta) + 2(1 - 2 \sin^2 \theta) - 2 = 0$

$\Rightarrow \quad 4 \sin^3 \theta - 4 \sin^2 \theta + 3 \sin \theta = 0$

$\Rightarrow \quad \sin \theta (4 \sin^2 \theta - 4 \sin \theta + 3) = 0$

$\Rightarrow \quad \sin \theta (4 \sin^2 \theta - 6 \sin \theta + 2 \sin \theta + 3) = 0$

$\Rightarrow \quad \sin \theta (2 \sin \theta + 1)(2 \sin \theta - 3) = 0$

$\Rightarrow \quad \sin \theta = 0 \text{ or } \sin \theta = -1/2 \text{ or } \sin \theta = 3/2$

$\Rightarrow \quad \theta = n\pi \text{ or } \theta = m\pi + (-1)^n \left(-\dfrac{\pi}{6} \right); m, n \in Z$

$\sin \theta = \dfrac{-3}{2}$ is not possible