9.9.2 The microscope

A simple magnifier or microscope is a converging lens of small focal length (Fig. 9.30). In order to use such a lens as a microscope, the lens is held near the object, one focal length away or less, and the eye is positioned close to the lens on the other side. The idea is to get an erect, magnified and virtual image of the object at a distance so that it can be viewed comfortably, i.e., at 25 cm or more. If the object is at a distance f, the image is at infinity. However, if the object is at a distance slightly less than the focal length of the lens, the image is virtual and closer than infinity. Although the closest comfortable distance for viewing the image is when it is at the near point (distance $D \cong 25$ cm), it causes some strain on the eye. Therefore, the image formed at infinity is often considered most suitable for viewing by the relaxed eye. We show both cases, the first in Fig. 9.30(a), and the second in Fig. 9.30(b) and (c).

The linear magnification m, for the image formed at the near point D, by a simple microscope can be obtained by using the relation

$$m \quad \frac{v}{u} \quad v \quad \frac{1}{v}-\frac{1}{f} \qquad 1-\frac{v}{f}$$

Now according to our sign convention, v is negative, and is equal in magnitude to D. Thus, the magnification is

$$m \quad 1 \quad \frac{D}{f} \tag{9.39}$$

Since D is about 25 cm, to have a magnification of six, one needs a convex lens of focal length, $f = 5$ cm.

Note that $m = h'/h$ where h is the size of the object and h' the size of the image. This is also the ratio of the angle subtended by the image to that subtended by the object, if placed at D for comfortable viewing. (Note that this is not the angle actually subtended by the object at the eye, which is h/u.) What a single-lens simple magnifier achieves is that it allows the object to be brought closer to the eye than D.

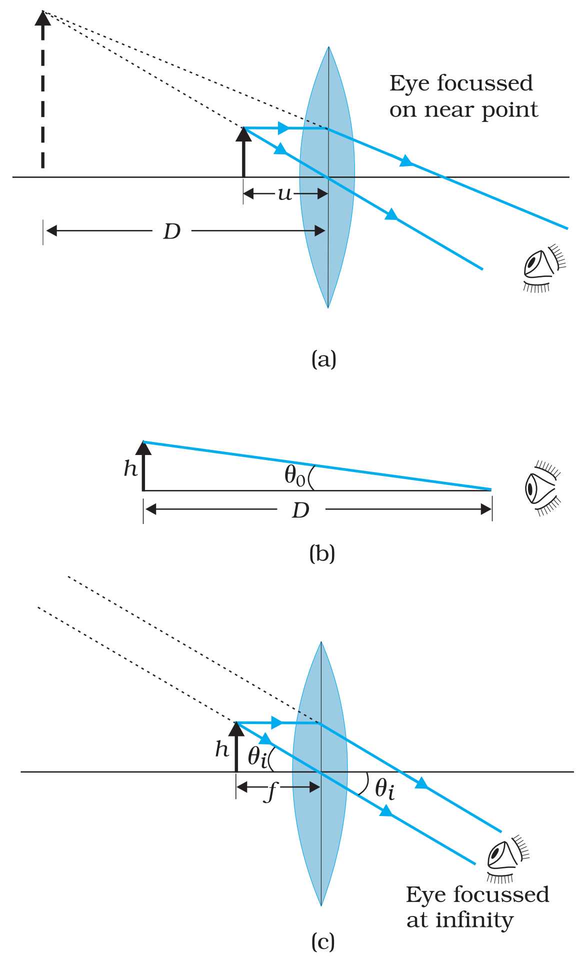

FIGURE 9.30 A simple microscope; (a) the magnifying lens is located such that the image is at the near point, (b) the angle subtended by the object, is the same as that at the near point, and (c) the object near the focal point of the lens; the image is far off but closer than infinity.

We will now find the magnification when the image is at infinity. In this case we will have to obtained the *angular* magnification. Suppose the object has a height h. The maximum angle it can subtend, and be clearly visible (without a lens), is when it is at the near point, i.e., a distance D. The angle subtended is then given by

$$\tan \quad {}_{o} \quad \frac{h}{D} \approx \theta_o \tag{9.40}$$

We now find the angle subtended at the eye by the image when the object is at u. From the relations

$$\frac{h}{h} \quad m \quad \frac{v}{u}$$

we have the angle subtended by the image

$\tan \ _i \quad \frac{h}{v} \quad \frac{h}{v}\frac{v}{u} \quad \frac{h}{u} \approx \theta$. The angle subtended by the object, when it is at $u = -f$.

$$_i \quad \frac{h}{f} \tag{9.41}$$

as is clear from Fig. 9.29(c). The angular magnification is, therefore

$$m \quad \frac{_i}{_o} \quad \frac{D}{f} \tag{9.42}$$

This is one less than the magnification when the image is at the near point, Eq. (9.39), but the viewing is more comfortable and the difference in magnification is usually small. In subsequent discussions of optical instruments (microscope and telescope) we shall assume the image to be at infinity.

A simple microscope has a limited maximum magnification (≤ 9) for realistic focal lengths. For much larger magnifications, one uses two lenses, one compounding the effect of the other. This is known as a *compound microscope*. A schematic diagram of a compound microscope is shown in Fig. 9.31. The lens nearest the object, called the *objective*, forms a real, inverted, magnified image of the object. This serves as the object for the second lens, the *eyepiece*, which functions essentially like a simple microscope or magnifier, produces the final image, which is enlarged and virtual. The first inverted image is thus near (at or within) the focal plane of the eyepiece, at a distance appropriate for final image formation at infinity, or a little closer for image formation at the near point. Clearly, the final image is inverted with respect to the original object.

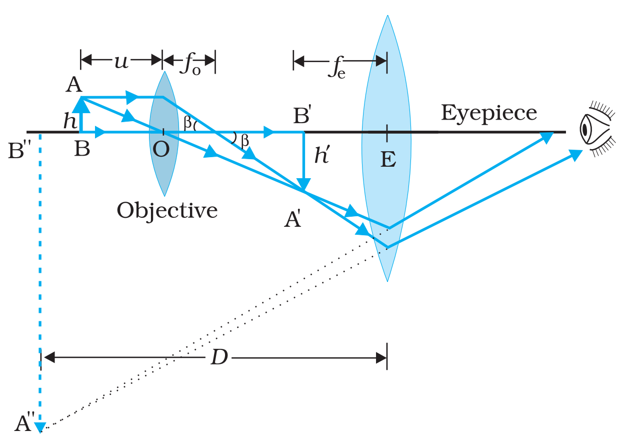

FIGURE 9.31 Ray diagram for the formation of image by a compound microscope.

We now obtain the magnification due to a compound microscope. The ray diagram of Fig. 9.31 shows that the (linear) magnification due to the objective, namely h'/h, equals

$$m_O \quad \frac{h}{h} \quad \frac{L}{f_o} \tag{9.43}$$

where we have used the result

$$\tan \quad \frac{h}{f_o} \quad \frac{h}{L}$$

Here h' is the size of the first image, the object size being h and f_o being the focal length of the objective. The first image is formed near the focal point of the eyepiece. The distance L, i.e., the distance between the second focal point of the objective and the first focal point of the eyepiece (focal length f_e) is called the tube length of the compound microscope.

As the first inverted image is near the focal point of the eyepiece, we use the result from the discussion above for the simple microscope to obtain the (angular) magnification m_e due to it [Eq. (9.39)], when the final image is formed at the near point, is

$$m_e \quad 1 \quad \frac{D}{f_e} \qquad [9.44(a)]$$

When the final image is formed at infinity, the angular magnification due to the eyepiece [Eq. (9.42)] is

$$m_e = (D/f_e) \qquad [9.44(b)]$$

Thus, the total magnification [(according to Eq. (9.33)], when the image is formed at infinity, is

$$m \quad m_o m_e \quad \frac{L}{f_o} \quad \frac{D}{f_e} \qquad (9.45)$$

Clearly, to achieve a large magnification of a *small* object (hence the name microscope), the objective and eyepiece should have small focal lengths. In practice, it is difficult to make the focal length much smaller than 1 cm. Also large lenses are required to make L large.

For example, with an objective with f_o = 1.0 cm, and an eyepiece with focal length f_e = 2.0 cm, and a tube length of 20 cm, the magnification is

$$m \quad m_o m_e \quad \frac{L}{f_o} \quad \frac{D}{f_e}$$

$$\frac{20}{1} \quad \frac{25}{2} \quad 250$$

Various other factors such as illumination of the object, contribute to the quality and visibility of the image. In modern microscopes, multi-component lenses are used for both the objective and the eyepiece to improve image quality by minimising various optical aberrations (defects) in lenses.

9.9.3 Telescope

The telescope is used to provide angular magnification of distant objects (Fig. 9.32). It also has an objective and an eyepiece. But here, the objective has a large focal length and a much larger aperture than the eyepiece. Light from a distant object enters the objective and a real image is formed in the tube at its second focal point. The eyepiece magnifies this image producing a final inverted image. The magnifying power m is the ratio of the angle β subtended at the eye by the final image to the angle α which the object subtends at the lens or the eye. Hence

$$m \quad — \quad \frac{h}{f_e} \cdot \frac{f_o}{h} \quad \frac{f_o}{f_e} \qquad (9.46)$$

In this case, the length of the telescope tube is $f_o + f_e$.

PHYSICS

The world's largest optical telescopes
http://astro.nineplanets.org/bigeyes.html

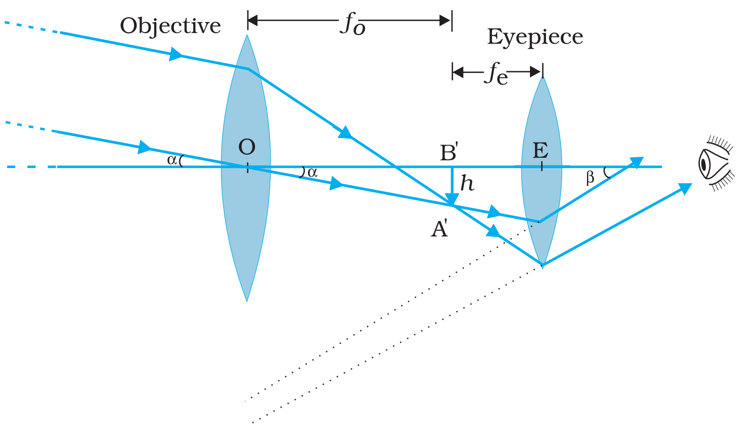

FIGURE 9.32 A refracting telescope.

Terrestrial telescopes have, in addition, a pair of inverting lenses to make the final image erect. Refracting telescopes can be used both for terrestrial and astronomical observations. For example, consider a telescope whose objective has a focal length of 100 cm and the eyepiece a focal length of 1 cm. The magnifying power of this telescope is m = 100/1 = 100.

Let us consider a pair of stars of actual separation 1′ (one minute of arc). The stars appear as though they are separated by an angle of 100 × 1′ = 100′ =1.67º.

The main considerations with an astronomical telescope are its light gathering power and its resolution or resolving power. The former clearly depends on the area of the objective. With larger diameters, fainter objects can be observed. The resolving power, or the ability to observe two objects distinctly, which are in very nearly the same direction, also depends on the diameter of the objective. So, the desirable aim in optical telescopes is to make them with objective of large diameter. The largest lens objective in use has a diameter of 40 inch (~1.02 m). It is at the Yerkes Observatory in Wisconsin, USA. Such big lenses tend to be very heavy and therefore, difficult to make and support by their edges. Further, it is rather difficult and expensive to make such large sized lenses which form images that are free from any kind of chromatic aberration and distortions.

For these reasons, modern telescopes use a concave mirror rather than a lens for the objective. Telescopes with mirror objectives are called *reflecting* telescopes. They have several advantages. First, there is no chromatic aberration in a mirror. Second, if a parabolic reflecting surface is chosen, spherical aberration is also removed. Mechanical support is much less of a problem since a mirror weighs much less than a lens of equivalent optical quality, and can be supported over its entire back surface, not just over its rim. One obvious problem with a reflecting telescope is that the objective mirror focusses light inside the telescope tube. One must have an eyepiece and the observer right there, obstructing some light (depending on the size of the observer cage). This is what is done in the very large 200 inch (~5.08 m) diameters, Mt. Palomar telescope, California. The viewer sits near the focal point of the mirror, in a small cage. Another solution to the problem is to deflect the light being focussed by another mirror. One such arrangement using a convex secondary mirror to focus the incident light, which now passes through a hole in the objective primary mirror, is shown

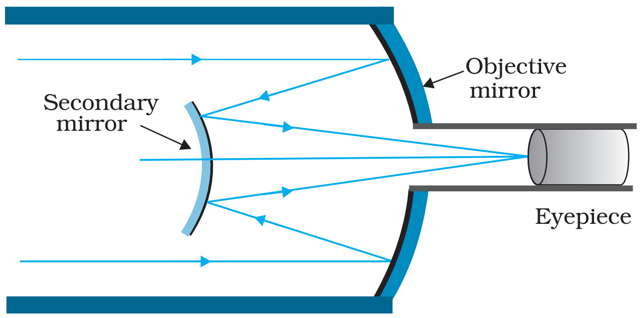

FIGURE 9.33 Schematic diagram of a reflecting telescope (Cassegrain).

in Fig. 9.33. This is known as a *Cassegrain* telescope, after its inventor. It has the advantages of a large focal length in a short telescope. The largest telescope in India is in Kavalur, Tamil Nadu. It is a 2.34 m diameter reflecting telescope (Cassegrain). It was ground, polished, set up, and is being used by the Indian Institute of Astrophysics, Bangalore. The largest reflecting telescopes in the world are the pair of Keck telescopes in Hawaii, USA, with a reflector of 10 metre in diameter.

SUMMARY

1. Reflection is governed by the equation $\angle i = \angle r'$ and refraction by the Snell's law, $\sin i/\sin r = n$, where the incident ray, reflected ray, refracted ray and normal lie in the same plane. Angles of incidence, reflection and refraction are i, r' and r, respectively.
2. The *critical angle of incidence* i_c for a ray incident from a denser to rarer medium, is that angle for which the angle of refraction is 90°. For $i > i_c$, total internal reflection occurs. Multiple internal reflections in diamond ($i_c \cong 24.4°$), totally reflecting prisms and mirage, are some examples of total internal reflection. Optical fibres consist of glass fibres coated with a thin layer of material of *lower* refractive index. Light incident at an angle at one end comes out at the other, after multiple internal reflections, even if the fibre is bent.
3. *Cartesian sign convention*: Distances measured in the same direction as the incident light are positive; those measured in the opposite direction are negative. All distances are measured from the pole/optic centre of the mirror/lens on the principal axis. The heights measured upwards above x-axis and normal to the principal axis of the mirror/lens are taken as positive. The heights measured downwards are taken as negative.
4. *Mirror equation*:

   $$\frac{1}{v} \quad \frac{1}{u} \quad \frac{1}{f}$$

   where u and v are object and image distances, respectively and f is the focal length of the mirror. f is (approximately) half the radius of curvature R. f is negative for concave mirror; f is positive for a convex mirror.
5. For a prism of the angle A, of refractive index n_2 placed in a medium of refractive index n_1,

   $$n_{21} \quad \frac{n_2}{n_1} \quad \frac{\sin \quad A \quad D_m \ /2}{\sin \ A/2}$$

   where D_m is the angle of minimum deviation.
6. *For refraction through a spherical interface* (from medium 1 to 2 of refractive index n_1 and n_2, respectively)

   $$\frac{n_2}{v} \quad \frac{n_1}{u} \quad \frac{n_2 \quad n_1}{R}$$

   Thin lens formula

   $$\frac{1}{v} \quad \frac{1}{u} \quad \frac{1}{f}$$