Substituting *i* and *r* from Eqs. (9.13) and (9.14), we get

$$\frac{n_1}{\text{OM}} \quad \frac{n_2}{\text{MI}} \quad \frac{n_2 \quad n_1}{\text{MC}} \qquad (9.15)$$

Here, OM, MI and MC represent magnitudes of distances. Applying the Cartesian sign convention,

OM = $-u$, MI = $+v$, MC = $+R$

Substituting these in Eq. (9.15), we get

$$\frac{n_2}{v} \quad \frac{n_1}{u} \quad \frac{n_2 \quad n_1}{R} \qquad (9.16)$$

Equation (9.16) gives us a relation between object and image distance in terms of refractive index of the medium and the radius of curvature of the curved spherical surface. It holds for any curved spherical surface.

Example 9.6 Light from a point source in air falls on a spherical glass surface (n = 1.5 and radius of curvature = 20 cm). The distance of the light source from the glass surface is 100 cm. At what position the image is formed?

Solution
We use the relation given by Eq. (9.16). Here
u = – 100 cm, v = ?, R = + 20 cm, n_1 = 1, and n_2 = 1.5.
We then have

$$\frac{1.5}{v} \quad \frac{1}{100} \quad \frac{0.5}{20}$$

or v = +100 cm
The image is formed at a distance of 100 cm from the glass surface, in the direction of incident light.

EXAMPLE 9.6

9.5.2 Refraction by a lens

Figure 9.18(a) shows the geometry of image formation by a double convex lens. The image formation can be seen in terms of two steps: (i) The first refracting surface forms the image I_1 of the object O [Fig. 9.18(b)]. The image I_1 acts as a virtual object for the second surface that forms the image at I [Fig. 9.18(c)]. Applying Eq. (9.15) to the first interface ABC, we get

$$\frac{n_1}{\text{OB}} \quad \frac{n_2}{\text{BI}_1} \quad \frac{n_2 \quad n_1}{\text{BC}_1} \qquad (9.17)$$

A similar procedure applied to the second interface* ADC gives,

$$\frac{n_2}{\text{DI}_1} \quad \frac{n_1}{\text{DI}} \quad \frac{n_2 \quad n_1}{\text{DC}_2} \qquad (9.18)$$

* Note that now the refractive index of the medium on the right side of ADC is n_1 while on its left it is n_2. Further DI_1 is negative as the distance is measured against the direction of incident light.

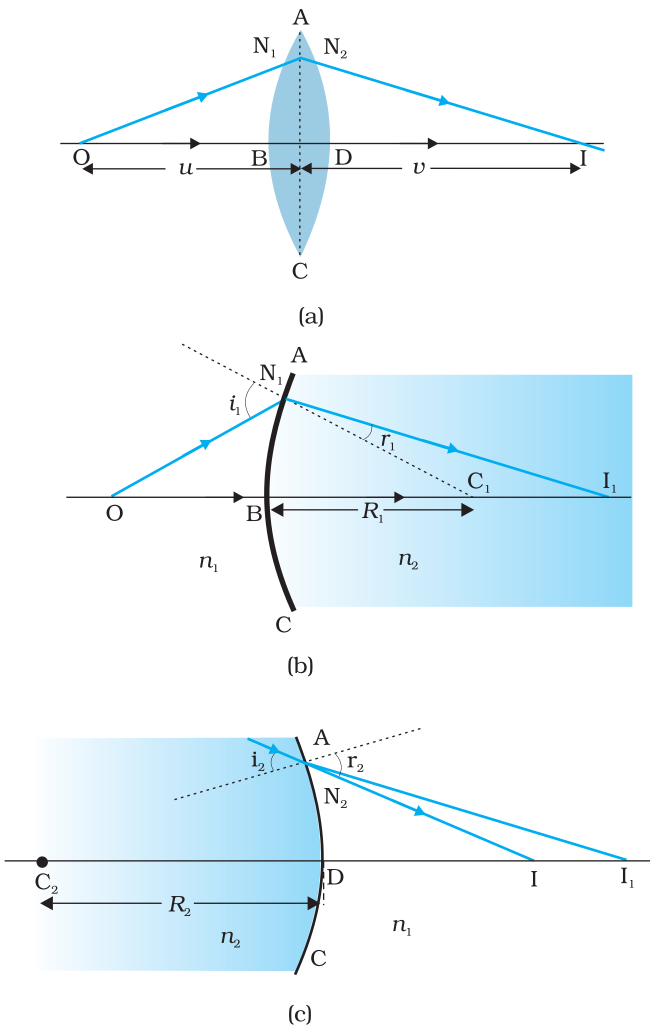

FIGURE 9.18 (a) The position of object, and the image formed by a double convex lens, (b) Refraction at the first spherical surface and (c) Refraction at the second spherical surface.

For a thin lens, $BI_1 = DI_1$. Adding Eqs. (9.17) and (9.18), we get

$$\frac{n_1}{OB} \quad \frac{n_1}{DI} \quad (n_2 \quad n_1) \quad \frac{1}{BC_1} \quad \frac{1}{DC_2} \tag{9.19}$$

Suppose the object is at infinity, i.e., $OB \rightarrow \infty$ and $DI = f$, Eq. (9.19) gives

$$\frac{n_1}{f} \quad (n_2 \quad n_1) \quad \frac{1}{BC_1} \quad \frac{1}{DC_2} \tag{9.20}$$

The point where image of an object placed at infinity is formed is called the *focus* F, of the lens and the distance f gives its *focal length*. A lens has two foci, F and F′, on either side of it (Fig. 9.19). By the sign convention,

$BC_1 = + R_1$,

$DC_2 = -R_2$

So Eq. (9.20) can be written as

$$\frac{1}{f} \quad n_{21} \quad 1 \quad \frac{1}{R_1} \quad \frac{1}{R_2} \qquad \because n_{21} \quad \frac{n_2}{n_1} \tag{9.21}$$

Equation (9.21) is known as the *lens maker's formula*. It is useful to design lenses of desired focal length using surfaces of suitable radii of curvature. Note that the formula is true for a concave lens also. In that case R_1 is negative, R_2 positive and therefore, f is negative.

From Eqs. (9.19) and (9.20), we get

$$\frac{n_1}{OB} \quad \frac{n_1}{DI} \quad \frac{n_1}{f} \tag{9.22}$$

Again, in the thin lens approximation, B and D are both close to the optical centre of the lens. Applying the sign convention,

$BO = - u$, $DI = +v$, we get

$$\frac{1}{v} \quad \frac{1}{u} \quad \frac{1}{f} \tag{9.23}$$

Equation (9.23) is the familiar *thin lens formula*. Though we derived it for a real image formed by a convex lens, the formula is valid for both convex as well as concave lenses and for both real and virtual images.

It is worth mentioning that the two foci, F and F′, of a double convex or concave lens are equidistant from the optical centre. The focus on the side of the (original) source of light is called the *first focal point*, whereas the other is called the *second focal point*.

To find the image of an object by a lens, we can, in principle, take any two rays emanating from a point on an object; trace their paths using the laws of refraction and find the point where the refracted rays meet (or appear to meet). In practice, however, it is convenient to choose any two of the following rays:

(i) A ray emanating from the object parallel to the principal axis of the lens after refraction passes through the second principal focus F′ (in a convex lens) or appears to diverge (in a concave lens) from the first principal focus F.

(ii) A ray of light, passing through the optical centre of the lens, emerges without any deviation after refraction.

(iii) A ray of light passing through the first principal focus (for a convex lens) or appearing to meet at it (for a concave lens) emerges parallel to the principal axis after refraction.

Figures 9.19(a) and (b) illustrate these rules for a convex and a concave lens, respectively. You should practice drawing similar ray diagrams for different positions of the object with respect to the lens and also verify that the lens formula, Eq. (9.23), holds good for all cases.

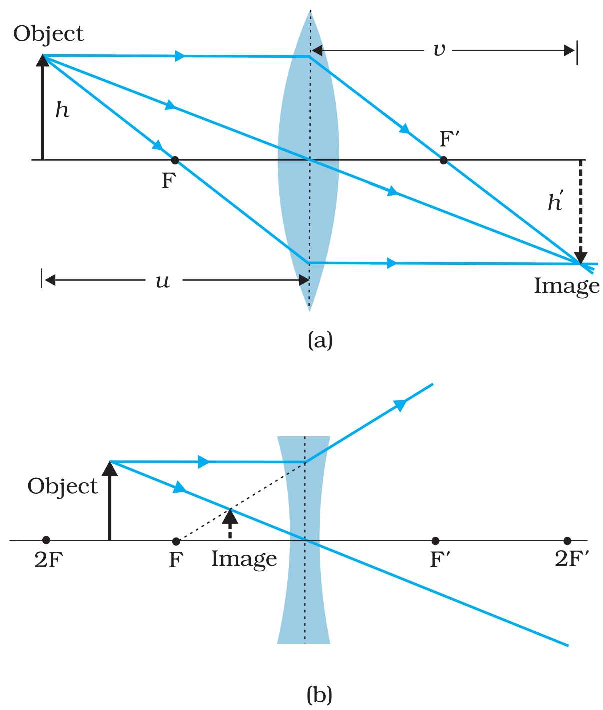

FIGURE 9.19 Tracing rays through (a) convex lens (b) concave lens.

Here again it must be remembered that each point on an object gives out infinite number of rays. All these rays will pass through the same image point after refraction at the lens.

Magnification (*m*) produced by a lens is defined, like that for a mirror, as the ratio of the size of the image to that of the object. Proceeding in the same way as for spherical mirrors, it is easily seen that for a lens

$$m = \frac{h'}{h} = \frac{v}{u} \quad (9.24)$$

When we apply the sign convention, we see that, for erect (and virtual) image formed by a convex or concave lens, *m* is positive, while for an inverted (and real) image, *m* is negative.

Example 9.7 A magician during a show makes a glass lens with n = 1.47 disappear in a trough of liquid. What is the refractive index of the liquid? Could the liquid be water?

Solution

The refractive index of the liquid must be equal to 1.47 in order to make the lens disappear. This means $n_1 = n_2$. This gives $1/f = 0$ or $f \to \infty$. The lens in the liquid will act like a plane sheet of glass. No, the liquid is not water. It could be glycerine.

EXAMPLE 9.7

9.5.3 Power of a lens

Power of a lens is a measure of the convergence or divergence, which a lens introduces in the light falling on it. Clearly, a lens of shorter focal

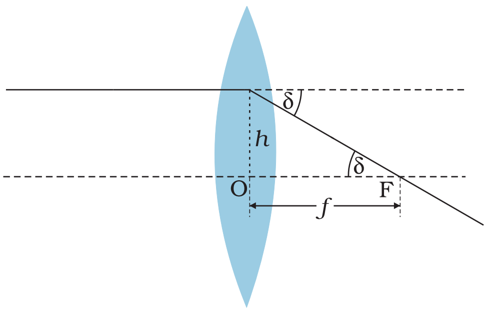

FIGURE 9.20 Power of a lens.

length bends the incident light more, while converging it in case of a convex lens and diverging it in case of a concave lens. The *power* P of a lens is defined as the tangent of the angle by which it converges or diverges a beam of light falling at unit distant from the optical centre (Fig. 9.20).

tan $\frac{h}{f}$; if h 1 tan $\frac{1}{f}$ or $\frac{1}{f}$ for small value of δ. Thus,

$$P = \frac{1}{f} \tag{9.25}$$

The SI unit for power of a lens is dioptre (D): 1D = 1m^{-1}. The power of a lens of focal length of 1 metre is one dioptre. Power of a lens is positive for a converging lens and negative for a diverging lens. Thus, when an optician prescribes a corrective lens of power + 2.5 D, the required lens is a convex lens of focal length + 40 cm. A lens of power of – 4.0 D means a concave lens of focal length – 25 cm.

Example 9.8 (i) If f = 0.5 m for a glass lens, what is the power of the lens? (ii) The radii of curvature of the faces of a double convex lens are 10 cm and 15 cm. Its focal length is 12 cm. What is the refractive index of glass? (iii) A convex lens has 20 cm focal length in air. What is focal length in water? (Refractive index of air-water = 1.33, refractive index for air-glass = 1.5.)

Solution

(i) Power = +2 dioptre.

(ii) Here, we have f = +12 cm, R_1 = +10 cm, R_2 = −15 cm.
Refractive index of air is taken as unity.
We use the lens formula of Eq. (9.22). The sign convention has to be applied for f, R_1 and R_2.
Substituting the values, we have

$$\frac{1}{12} \quad (n \quad 1) \quad \frac{1}{10} \quad \frac{1}{15}$$

This gives n = 1.5.

(iii) For a glass lens in air, n_2 = 1.5, n_1 = 1, f = +20 cm. Hence, the lens formula gives

$$\frac{1}{20} \quad 0.5 \quad \frac{1}{R_1} \quad \frac{1}{R_2}$$

For the same glass lens in water, n_2 = 1.5, n_1 = 1.33. Therefore,

$$\frac{1.33}{f} \quad (1.5 \quad 1.33) \quad \frac{1}{R_1} \quad \frac{1}{R_2} \tag{9.26}$$

Combining these two equations, we find f = + 78.2 cm.

EXAMPLE 9.8

9.5.4 Combination of thin lenses in contact

Consider two lenses A and B of focal length f_1 and f_2 placed in contact with each other. Let the object be placed at a point O beyond the focus of the first lens A (Fig. 9.21). The first lens produces an image at I_1. Since image I_1 is real, it serves as a virtual object for the second lens B, producing the final image at I. It must, however, be borne in mind that formation of image by the first lens is presumed only to facilitate determination of the position of the final image. In fact, the direction of rays emerging from the first lens gets modified in accordance with the angle at which they strike the second lens. Since the lenses are thin, we assume the optical centres of the lenses to be coincident. Let this central point be denoted by P.

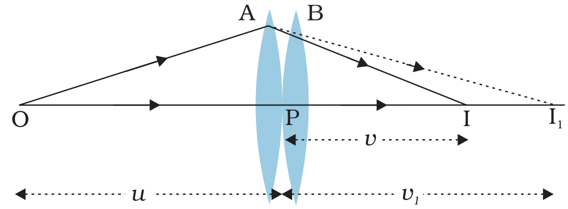

FIGURE 9.21 Image formation by a combination of two thin lenses in contact.

For the image formed by the first lens A, we get

$$\frac{1}{v_1} \quad \frac{1}{u} \quad \frac{1}{f_1} \tag{9.27}$$

For the image formed by the second lens B, we get

$$\frac{1}{v} \quad \frac{1}{v_1} \quad \frac{1}{f_2} \tag{9.28}$$

Adding Eqs. (9.27) and (9.28), we get

$$\frac{1}{v} \quad \frac{1}{u} \quad \frac{1}{f_1} \quad \frac{1}{f_2} \tag{9.29}$$

If the two lens-system is regarded as equivalent to a single lens of focal length f, we have

$$\frac{1}{v} \quad \frac{1}{u} \quad \frac{1}{f}$$

so that we get

$$\frac{1}{f} \quad \frac{1}{f_1} \quad \frac{1}{f_2} \tag{9.30}$$

The derivation is valid for any number of thin lenses in contact. If several thin lenses of focal length $f_1, f_2, f_3, \ldots$ are in contact, the effective focal length of their combination is given by

$$\frac{1}{f} \quad \frac{1}{f_1} \quad \frac{1}{f_2} \quad \frac{1}{f_3} \quad \ldots \tag{9.31}$$

In terms of power, Eq. (9.31) can be written as

$$P = P_1 + P_2 + P_3 + \ldots \tag{9.32}$$

where P is the net power of the lens combination. Note that the sum in Eq. (9.32) is an algebraic sum of individual powers, so some of the terms on the right side may be positive (for convex lenses) and some negative (for concave lenses). Combination of lenses helps to obtain diverging or converging lenses of desired magnification. It also enhances sharpness of the image. Since the image formed by the first lens becomes the object for the second, Eq. (9.25) implies that the total magnification m of the combination is a product of magnification (m_1, m_2, m_3,...) of individual lenses

$$m = m_1\, m_2\, m_3\ \ldots \tag{9.33}$$

Such a system of combination of lenses is commonly used in designing lenses for cameras, microscopes, telescopes and other optical instruments.

Example 9.9 Find the position of the image formed by the lens combination given in the Fig. 9.22.

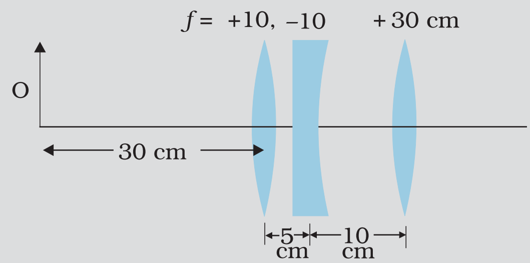

FIGURE 9.22

Solution Image formed by the first lens

$$\frac{1}{v_1} \quad \frac{1}{u_1} \quad \frac{1}{f_1}$$

$$\frac{1}{v_1} \quad \frac{1}{\ \ 30} \quad \frac{1}{10}$$

or $v_1 = 15$ cm

The image formed by the first lens serves as the object for the second. This is at a distance of (15 – 5) cm = 10 cm to the right of the second lens. Though the image is real, it serves as a virtual object for the second lens, which means that the rays appear to come from it for the second lens.

$$\frac{1}{v_2} \quad \frac{1}{10} \quad \frac{1}{\ \ 10}$$

or $v_2 = \infty$

The virtual image is formed at an infinite distance to the left of the second lens. This acts as an object for the third lens.

$$\frac{1}{v_3} \quad \frac{1}{u_3} \quad \frac{1}{f_3}$$

or $\frac{1}{v_3} \quad \frac{1}{} \quad \frac{1}{30}$

or $v_3 = 30$ cm

The final image is formed 30 cm to the right of the third lens.

EXAMPLE 9.9

9.6 Refraction through a Prism

Figure 9.23 shows the passage of light through a triangular prism ABC. The angles of incidence and refraction at the first face AB are i and r_1, while the angle of incidence (from glass to air) at the second face AC is r_2 and the angle of refraction or emergence e. The angle between the emergent ray RS and the direction of the incident ray PQ is called the *angle of deviation*, δ.