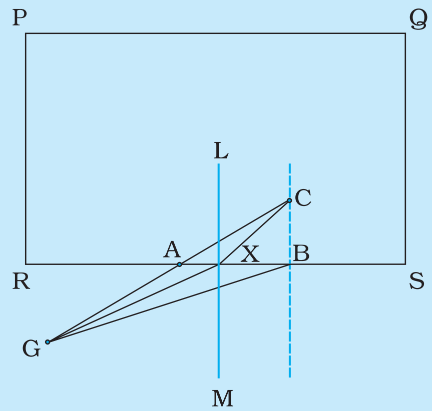
THE DROWNING CHILD, LIFEGUARD AND SNELL'S LAW

Consider a rectangular swimming pool PQSR; see figure here. A lifeguard sitting at G outside the pool notices a child drowning at a point C. The guard wants to reach the child in the shortest possible time. Let SR be the side of the pool between G and C. Should he/she take a straight line path GAC between G and C or GBC in which the path BC in water would be the shortest, or some other path GXC? The guard knows that his/her running speed v_1 on ground is higher than his/her swimming speed v_2.



Suppose the guard enters water at X. Let GX $= l_1$ and XC $= l_2$. Then the time taken to reach from G to C would be

$$t \quad \frac{l_1}{v_1} \quad \frac{l_2}{v_2}$$

To make this time minimum, one has to differentiate it (with respect to the coordinate of X) and find the point X when t is a minimum. On doing all this algebra (which we skip here), we find that the guard should enter water at a point where Snell's law is satisfied. To understand this, draw a perpendicular LM to side SR at X. Let $\angle$GXM = i and $\angle$CXL = r. Then it can be seen that t is minimum when

$$\frac{\sin i}{\sin r} \quad \frac{v_1}{v_2}$$

In the case of light v_1/v_2, the ratio of the velocity of light in vacuum to that in the medium, is the refractive index n of the medium.

In short, whether it is a wave or a particle or a human being, whenever two mediums and two velocities are involved, one must follow Snell's law if one wants to take the shortest time.

9.4 TOTAL INTERNAL REFLECTION

When light travels from an optically denser medium to a rarer medium at the interface, it is partly reflected back into the same medium and partly refracted to the second medium. This reflection is called the *internal reflection.*

When a ray of light enters from a denser medium to a rarer medium, it bends away from the normal, for example, the ray AO_1B in Fig. 9.12. The incident ray AO_1 is partially reflected (O_1C) and partially transmitted (O_1B) or refracted, the angle of refraction (r) being larger than the angle of incidence (i). As the angle of incidence increases, so does the angle of refraction, till for the ray AO_3, the angle of refraction is $\pi/2$. The refracted ray is bent so much away from the normal that it grazes the surface at the interface between the two media. This is shown by the ray AO_3D in Fig. 9.12. If the angle of incidence is increased still further (e.g., the ray AO_4), refraction is not possible, and the incident ray is totally reflected.

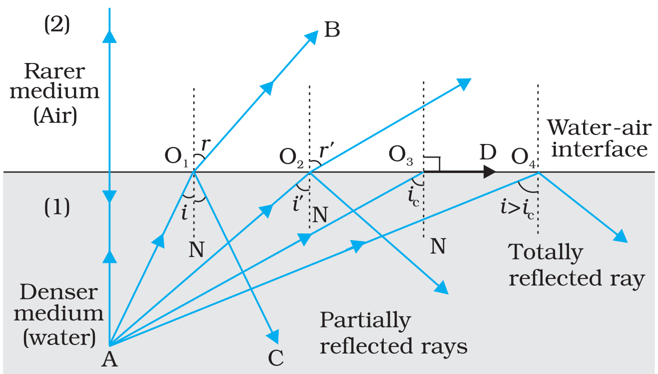


FIGURE 9.12 Refraction and internal reflection of rays from a point A in the denser medium (water) incident at different angles at the interface with a rarer medium (air).

This is called *total internal reflection*. When light gets reflected by a surface, normally some fraction of it gets transmitted. The reflected ray, therefore, is always less intense than the incident ray, howsoever smooth the reflecting surface may be. In total internal reflection, on the other hand, no transmission of light takes place.

The angle of incidence corresponding to an angle of refraction 90º, say $\angle AO_3N$, is called the *critical angle* (i_c) for the given pair of media. We see from Snell's law [Eq. (9.10)] that if the relative refractive index is less than one then, since the maximum value of sin r is unity, there is an upper limit to the value of sin i for which the law can be satisfied, that is, $i = i_c$ such that

$$\sin i_c = n_{21} \tag{9.12}$$

For values of i larger than i_c, Snell's law of refraction cannot be satisfied, and hence no refraction is possible.

The refractive index of denser medium 1 with respect to rarer medium 2 will be $n_{12} = 1/\sin i_c$. Some typical critical angles are listed in Table 9.1.

TABLE 9.1 CRITICAL ANGLE OF SOME TRANSPARENT MEDIA WITH RESPECT TO AIR

Substance medium	Refractive index	Critical angle
Water	1.33	48.75°
Crown glass	1.52	41.14°
Dense flint glass	1.62	37.31°
Diamond	2.42	24.41°

A demonstration for total internal reflection

All optical phenomena can be demonstrated very easily with the use of a laser torch or pointer, which is easily available nowadays. Take a glass beaker with clear water in it. Stir the water a few times with a piece of soap, so that it becomes a little turbid. Take a laser pointer and shine its beam through the turbid water. You will find that the path of the beam inside the water shines brightly.

Shine the beam from below the beaker such that it strikes at the upper water surface at the other end. Do you find that it undergoes partial reflection (which is seen as a spot on the table below) and partial refraction [which comes out in the air and is seen as a spot on the roof; Fig. 9.13(a)]? Now direct the laser beam from one side of the beaker such that it strikes the upper surface of water more obliquely [Fig. 9.13(b)]. Adjust the direction of laser beam until you find the angle for which the refraction above the water surface is totally absent and the beam is totally reflected back to water. This is total internal reflection at its simplest.

Pour this water in a long test tube and shine the laser light from top, as shown in Fig. 9.13(c). Adjust the direction of the laser beam such that it is totally internally reflected every time it strikes the walls of the tube. This is similar to what happens in optical fibres.

Take care not to look into the laser beam directly and not to point it at anybody's face.

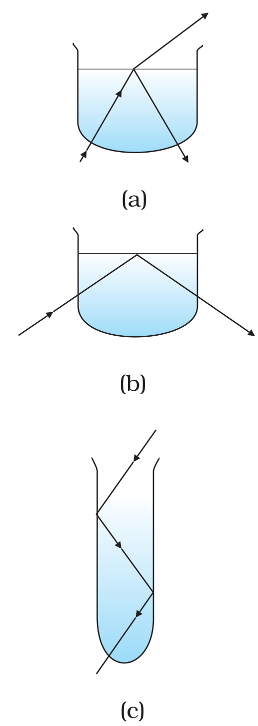


FIGURE 9.13 Observing total internal reflection in water with a laser beam (refraction due to glass of beaker neglected being very thin).

9.4.1 Total internal reflection in nature and its technological applications

(i) *Mirage:* On hot summer days, the air near the ground becomes hotter than the air at higher levels. The refractive index of air increases with its density. Hotter air is less dense, and has smaller refractive index than the cooler air. If the air currents are small, that is, the air is still, the optical density at different layers of air increases with height. As a result, light from a tall object such as a tree, passes through a medium whose refractive index decreases towards the ground. Thus, a ray of light from such an object successively bends away from the normal and undergoes total internal reflection, if the angle of incidence for the air near the ground exceeds the critical angle. This is shown in Fig. 9.14(b). To a distant observer, the light appears to be coming from somewhere below the ground. The observer naturally assumes that light is being reflected from the ground, say, by a pool of water near the tall object. Such inverted images of distant tall objects cause an optical illusion to the observer. This phenomenon is called *mirage*. This type of mirage is especially common in hot deserts. Some of you might have noticed that while moving in a bus or a car during a hot summer day, a distant patch of road, especially on a highway, appears to be wet. But, you do not find any evidence of wetness when you reach that spot. This is also due to mirage.

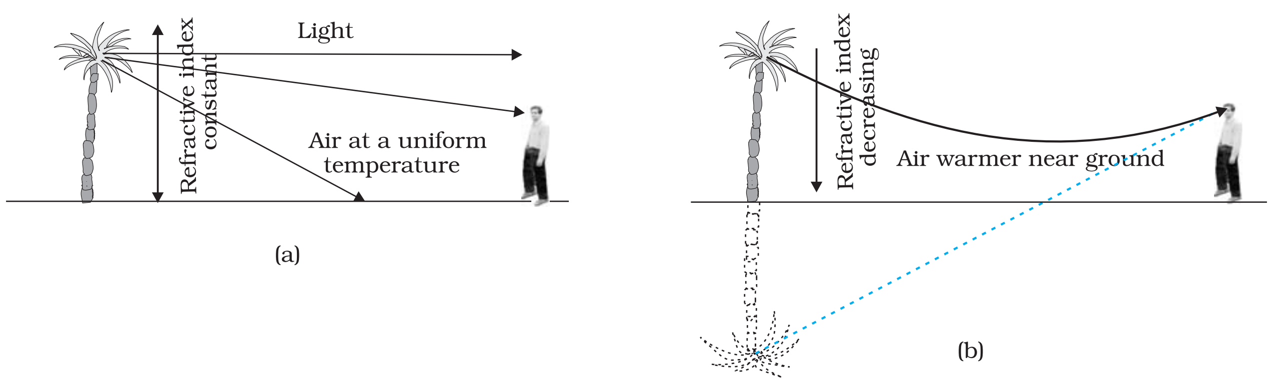


FIGURE 9.14 (a) A tree is seen by an observer at its place when the air above the ground is at uniform temperature, (b) When the layers of air close to the ground have varying temperature with hottest layers near the ground, light from a distant tree may undergo total internal reflection, and the apparent image of the tree may create an illusion to the observer that the tree is near a pool of water.

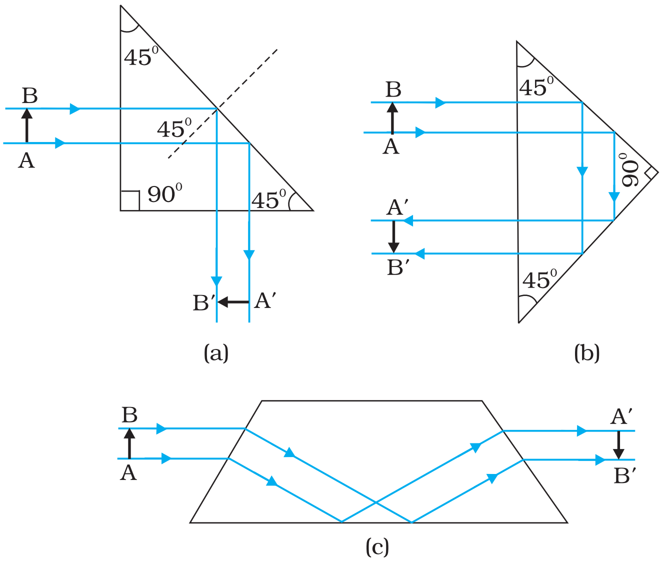


FIGURE 9.15 Prisms designed to bend rays by 90º and 180º or to invert image without changing its size make use of total internal reflection.

(ii) *Diamond*: Diamonds are known for their spectacular brilliance. Their brilliance is mainly due to the total internal reflection of light inside them. The critical angle for diamond-air interface (≅ 24.4°) is very small, therefore once light enters a diamond, it is very likely to undergo total internal reflection inside it. Diamonds found in nature rarely exhibit the brilliance for which they are known. It is the technical skill of a diamond cutter which makes diamonds to sparkle so brilliantly. By cutting the diamond suitably, multiple total internal reflections can be made to occur.

(iii) *Prism*: Prisms designed to bend light by 90º or by 180º make use of total internal reflection [Fig. 9.15(a) and (b)]. Such a prism is also used to invert images without changing their size [Fig. 9.15(c)].

In the first two cases, the critical angle i_c for the material of the prism must be less than 45º. We see from Table 9.1 that this is true for both crown glass and dense flint glass.

(iv) *Optical fibres*: Now-a-days optical fibres are extensively used for transmitting audio and video signals through long distances. Optical fibres too make use of the phenomenon of total internal reflection. Optical fibres are fabricated with high quality composite glass/quartz fibres. Each fibre consists of a core and cladding. The refractive index of the material of the core is higher than that of the cladding.

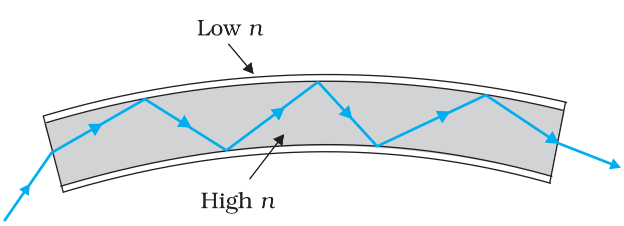


FIGURE 9.16 Light undergoes successive total internal reflections as it moves through an optical fibre.

When a signal in the form of light is directed at one end of the fibre at a suitable angle, it undergoes repeated total internal reflections along the length of the fibre and finally comes out at the other end (Fig. 9.16). Since light undergoes total internal reflection at each stage, there is no appreciable loss in the intensity of the light signal. Optical fibres are fabricated such that light reflected at one side of inner surface strikes the other at an angle larger than the critical angle. Even if the fibre is bent, light can easily travel along its length. Thus, an optical fibre can be used to act as an optical pipe.

A bundle of optical fibres can be put to several uses. Optical fibres are extensively used for transmitting and receiving electrical signals which are converted to light by suitable transducers. Obviously, optical fibres can also be used for transmission of optical signals. For example, these are used as a 'light pipe' to facilitate visual examination of internal organs like esophagus, stomach and intestines. You might have seen a commonly available decorative lamp with fine plastic fibres with their free ends forming a fountain like structure. The other end of the fibres is fixed over an electric lamp. When the lamp is switched on, the light travels from the bottom of each fibre and appears at the tip of its free end as a dot of light. The fibres in such decorative lamps are optical fibres.

The main requirement in fabricating optical fibres is that there should be very little absorption of light as it travels for long distances inside them. This has been achieved by purification and special preparation of materials such as quartz. In silica glass fibres, it is possible to transmit more than 95% of the light over a fibre length of 1 km. (Compare with what you expect for a block of ordinary window glass 1 km thick.)

9.5 Refraction at Spherical Surfaces and by Lenses

We have so far considered refraction at a plane interface. We shall now consider refraction at a spherical interface between two transparent media. An infinitesimal part of a spherical surface can be regarded as planar and the same laws of refraction can be applied at every point on the surface. Just as for reflection by a spherical mirror, the normal at the point of incidence is perpendicular to the tangent plane to the spherical surface at that point and, therefore, passes through its centre of curvature. We first consider refraction by a single spherical surface and follow it by thin lenses. A thin lens is a transparent optical medium bounded by two surfaces; at least one of which should be spherical. Applying the formula for image formation by a single spherical surface successively at the two surfaces of a lens, we shall obtain the lens maker's formula and then the lens formula.

9.5.1 Refraction at a spherical surface

Figure 9.17 shows the geometry of formation of image *I* of an object *O* on the principal axis of a spherical surface with centre of curvature C, and radius of curvature *R*. The rays are incident from a medium of refractive index n_1, to another of refractive index n_2. As before, we take the aperture (or the lateral size) of the surface to be small compared to other distances involved, so that small angle approximation can be made. In particular, NM will be taken to be nearly equal to the length of the perpendicular from the point N on the principal axis. We have, for small angles,

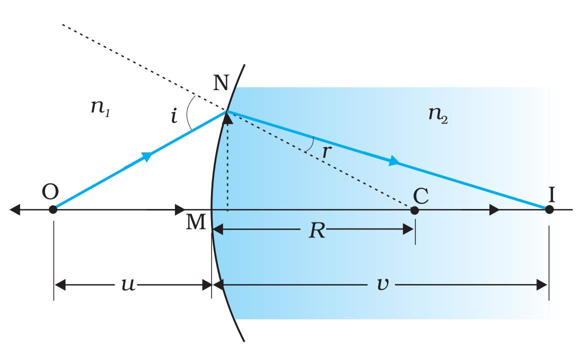


FIGURE 9.17 Refraction at a spherical surface separating two media.

$$\tan \angle NOM = \frac{MN}{OM}$$

$$\tan \angle NCM = \frac{MN}{MC}$$

$$\tan \angle NIM = \frac{MN}{MI}$$