Chapter Nine

RAY OPTICS AND OPTICAL INSTRUMENTS

9.1 Introduction

Nature has endowed the human eye (retina) with the sensitivity to detect electromagnetic waves within a small range of the electromagnetic spectrum. Electromagnetic radiation belonging to this region of the spectrum (wavelength of about 400 nm to 750 nm) is called light. It is mainly through light and the sense of vision that we know and interpret the world around us.

There are two things that we can intuitively mention about light from common experience. First, that it travels with enormous speed and second, that it travels in a straight line. It took some time for people to realise that the speed of light is finite and measurable. Its presently accepted value in vacuum is $c = 2.99792458 \times 10^8$ m s^{-1}. For many purposes, it suffices to take $c = 3 \times 10^8$ m s^{-1}. The speed of light in vacuum is the highest speed attainable in nature.

The intuitive notion that light travels in a straight line seems to contradict what we have learnt in Chapter 8, that light is an electromagnetic wave of wavelength belonging to the visible part of the spectrum. How to reconcile the two facts? The answer is that the wavelength of light is very small compared to the size of ordinary objects that we encounter commonly (generally of the order of a few cm or larger). In this situation, as you will learn in Chapter 10, a light wave can be considered to travel from one point to another, along a straight line joining them. The path is called a *ray* of light, and a bundle of such rays constitutes a *beam* of light.

In this chapter, we consider the phenomena of reflection, refraction and dispersion of light, using the ray picture of light. Using the basic laws of reflection and refraction, we shall study the image formation by plane and spherical reflecting and refracting surfaces. We then go on to describe the construction and working of some important optical instruments, including the human eye.

PARTICLE MODEL OF LIGHT

Newton's fundamental contributions to mathematics, mechanics, and gravitation often blind us to his deep experimental and theoretical study of light. He made pioneering contributions in the field of optics. He further developed the corpuscular model of light proposed by Descartes. It presumes that light energy is concentrated in tiny particles called *corpuscles*. He further assumed that corpuscles of light were massless elastic particles. With his understanding of mechanics, he could come up with a simple model of reflection and refraction. It is a common observation that a ball bouncing from a smooth plane surface obeys the laws of reflection. When this is an elastic collision, the magnitude of the velocity remains the same. As the surface is smooth, there is no force acting parallel to the surface, so the component of momentum in this direction also remains the same. Only the component perpendicular to the surface, i.e., the normal component of the momentum, gets reversed in reflection. Newton argued that smooth surfaces like mirrors reflect the corpuscles in a similar manner.

In order to explain the phenomena of refraction, Newton postulated that the speed of the corpuscles was greater in water or glass than in air. However, later on it was discovered that the speed of light is less in water or glass than in air.

In the field of optics, Newton – the experimenter, was greater than Newton – the theorist. He himself observed many phenomena, which were difficult to understand in terms of particle nature of light. For example, the colours observed due to a thin film of oil on water. Property of partial reflection of light is yet another such example. Everyone who has looked into the water in a pond sees image of the face in it, but also sees the bottom of the pond. Newton argued that some of the corpuscles, which fall on the water, get reflected and some get transmitted. But what property could distinguish these two kinds of corpuscles? Newton had to postulate some kind of unpredictable, chance phenomenon, which decided whether an individual corpuscle would be reflected or not. In explaining other phenomena, however, the corpuscles were presumed to behave as if they are identical. Such a dilemma does not occur in the wave picture of light. An incoming wave can be divided into two weaker waves at the boundary between air and water.

9.2 REFLECTION OF LIGHT BY SPHERICAL MIRRORS

We are familiar with the laws of reflection. The angle of reflection (i.e., the angle between reflected ray and the normal to the reflecting surface or the mirror) equals the angle of incidence (angle between incident ray and the normal). Also that the incident ray, reflected ray and the normal to the reflecting surface at the point of incidence lie in the same plane (Fig. 9.1). These laws are valid at each point on any reflecting surface whether plane or curved. However, we shall restrict our discussion to the special case of curved surfaces, that is, spherical surfaces. The normal in this case is to be taken as normal to the tangent to surface at the point of incidence. That is, the normal is along the radius, the line joining the centre of curvature of the mirror to the point of incidence.

We have already studied that the geometric centre of a spherical mirror is called its pole while that of a spherical lens is called its optical centre. The line joining the pole and the centre of curvature of the spherical mirror is known as the *principal axis*. In the case of spherical lenses, the principal axis is the line joining the optical centre with its principal focus as you will see later.

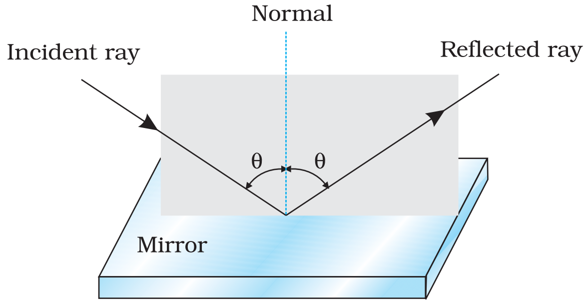

FIGURE 9.1 The incident ray, reflected ray and the normal to the reflecting surface lie in the same plane.

9.2.1 Sign convention

To derive the relevant formulae for reflection by spherical mirrors and refraction by spherical lenses, we must first adopt a sign convention for measuring distances. In this book, we shall follow the *Cartesian sign convention*. According to this convention, all distances are measured from the pole of the mirror or the optical centre of the lens. The distances measured in the same direction as the incident light are taken as positive and those measured in the direction opposite to the direction of incident light are taken as negative (Fig. 9.2). The heights measured upwards with respect to *x*-axis and normal to the principal axis (*x*-axis) of the mirror/lens are taken as positive (Fig. 9.2). The heights measured downwards are taken as negative.

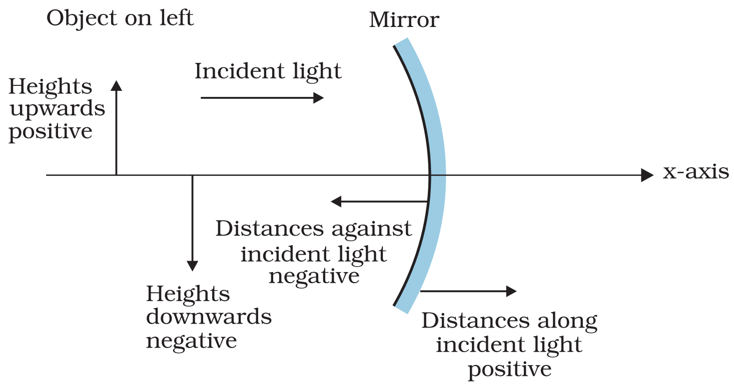

FIGURE 9.2 The Cartesian Sign Convention.

With a common accepted convention, it turns out that a single formula for spherical mirrors and a single formula for spherical lenses can handle all different cases.

9.2.2 Focal length of spherical mirrors

Figure 9.3 shows what happens when a parallel beam of light is incident on (a) a concave mirror, and (b) a convex mirror. We assume that the rays are *paraxial*, i.e., they are incident at points close to the pole P of the mirror and make small angles with the principal axis. The reflected rays converge at a point F on the principal axis of a concave mirror [Fig. 9.3(a)]. For a convex mirror, the reflected rays appear to diverge from a point F on its principal axis [Fig. 9.3(b)]. The point F is called the *principal focus* of the mirror. If the parallel paraxial beam of light were incident, making some angle with the principal axis, the reflected rays would converge (or appear to diverge) from a point in a plane through F normal to the principal axis. This is called the *focal plane* of the mirror [Fig. 9.3(c)].

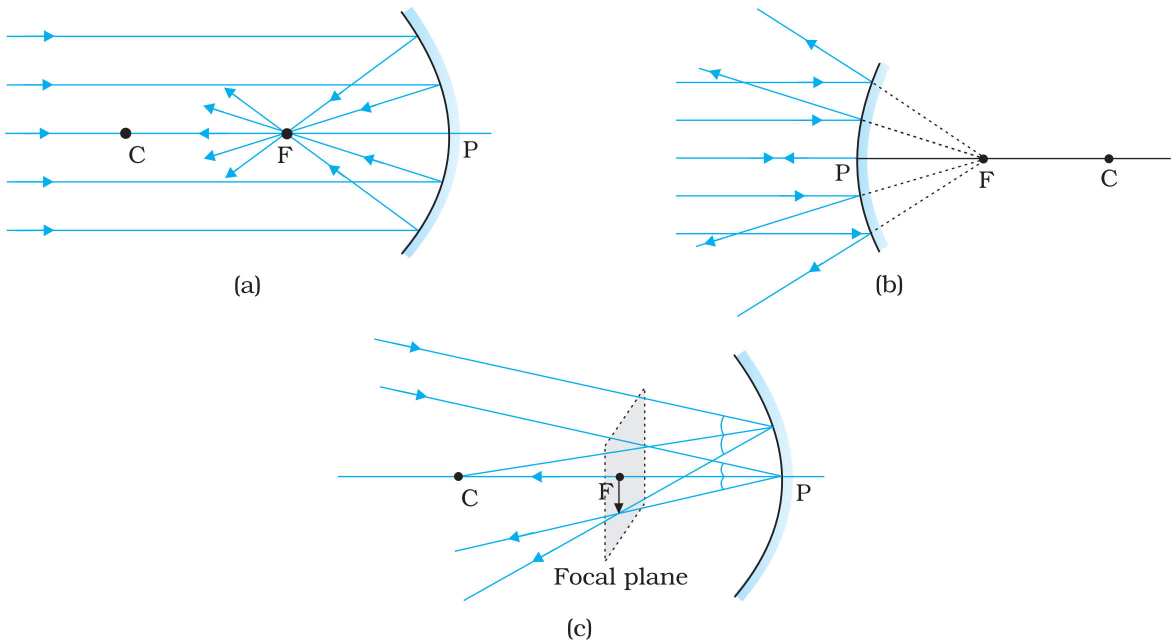

FIGURE 9.3 Focus of a concave and convex mirror.

The distance between the focus F and the pole P of the mirror is called the *focal length* of the mirror, denoted by f. We now show that $f = R/2$, where R is the radius of curvature of the mirror. The geometry of reflection of an incident ray is shown in Fig. 9.4.

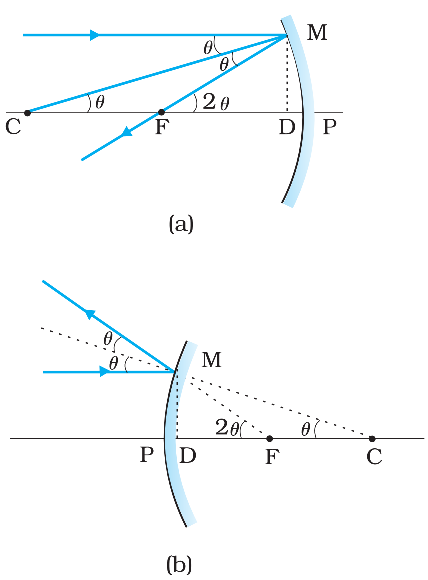

FIGURE 9.4 Geometry of reflection of an incident ray on (a) concave spherical mirror, and (b) convex spherical mirror.

Let C be the centre of curvature of the mirror. Consider a ray parallel to the principal axis striking the mirror at M. Then CM will be perpendicular to the mirror at M. Let θ be the angle of incidence, and MD be the perpendicular from M on the principal axis. Then,

$$\angle MCP = \theta \text{ and } \angle MFP = 2\theta$$

Now,

$$\tan\theta = \frac{MD}{CD} \text{ and } \tan 2\theta = \frac{MD}{FD} \qquad (9.1)$$

For small θ, which is true for paraxial rays, $\tan\theta \approx \theta$, $\tan 2\theta \approx 2\theta$. Therefore, Eq. (9.1) gives

$$\frac{MD}{FD} = 2\frac{MD}{CD}$$

$$\text{or, } FD = \frac{CD}{2} \qquad (9.2)$$

Now, for small θ, the point D is very close to the point P. Therefore, FD = f and CD = R. Equation (9.2) then gives

$$f = R/2 \qquad (9.3)$$

9.2.3 The mirror equation

If rays emanating from a point actually meet at another point after reflection and/or refraction, that point is called the *image* of the first point. The image is *real* if the rays actually converge to the point; it is *virtual* if the rays do not actually meet but appear to diverge from the point when produced backwards. An image is thus a point-to-point correspondence with the object established through reflection and/or refraction.

In principle, we can take any two rays emanating from a point on an object, trace their paths, find their point of intersection and thus, obtain the image of the point due to reflection at a spherical mirror. In practice, however, it is convenient to choose any two of the following rays:

(i) The ray from the point which is parallel to the principal axis. The reflected ray goes through the focus of the mirror.

(ii) The ray passing through the centre of curvature of a concave mirror or appearing to pass through it for a convex mirror. The reflected ray simply retraces the path.

(iii) The ray passing through (or directed towards) the focus of the concave mirror or appearing to pass through (or directed towards) the focus of a convex mirror. The reflected ray is parallel to the principal axis.

(iv) The ray incident at any angle at the pole. The reflected ray follows laws of reflection.

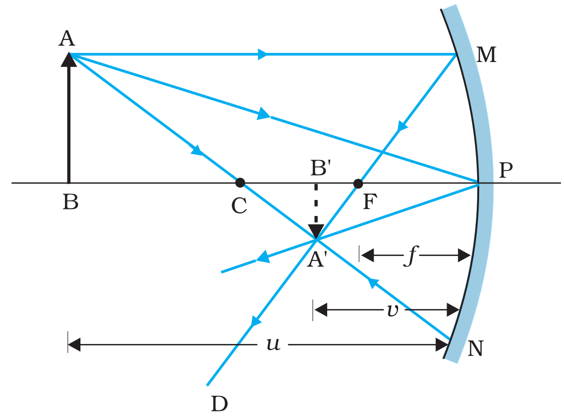

FIGURE 9.5 Ray diagram for image formation by a concave mirror.

Figure 9.5 shows the ray diagram considering three rays. It shows the image A′B′ (in this case, real) of an object AB formed by a concave mirror. It does not mean that only three rays emanate from the point A. An infinite number of rays emanate from any source, in all directions. Thus, point A′ is image point of A if every ray originating at point A and falling on the concave mirror after reflection passes through the point A′.

We now derive the mirror equation or the relation between the object distance (u), image distance (v) and the focal length (f).

From Fig. 9.5, the two right-angled triangles A′B′F and MPF are similar. (For paraxial rays, MP can be considered to be a straight line perpendicular to CP.) Therefore,

$$\frac{BA}{PM} \quad \frac{BF}{FP}$$

$$\text{or} \quad \frac{BA}{BA} \quad \frac{BF}{FP} \quad (\because PM = AB) \qquad (9.4)$$

Since $\angle$ APB = $\angle$ A′PB′, the right angled triangles A′B′P and ABP are also similar. Therefore,

$$\frac{BA}{BA} \quad \frac{BP}{BP} \qquad (9.5)$$

Comparing Eqs. (9.4) and (9.5), we get

$$\frac{BF}{FP} \quad \frac{BP - FP}{FP} \quad \frac{BP}{BP} \qquad (9.6)$$

Equation (9.6) is a relation involving magnitude of distances. We now apply the sign convention. We note that light travels from the object to the mirror MPN. Hence this is taken as the positive direction. To reach the object AB, image A′B′ as well as the focus F from the pole P, we have to travel opposite to the direction of incident light. Hence, all the three will have negative signs. Thus,

B′ P = $-v$, FP = $-f$, BP = $-u$

Using these in Eq. (9.6), we get

$$\frac{-v \quad f}{-f} \qquad \frac{-v}{-u}$$

or $\frac{v - f}{f} \qquad \frac{v}{u}$

$$\frac{1}{v} \quad \frac{1}{u} \qquad \frac{1}{f} \qquad (9.7)$$

This relation is known as the *mirror equation*.

The size of the image relative to the size of the object is another important quantity to consider. We define linear *magnification* (m) as the ratio of the height of the image (h') to the height of the object (h):

$$m = \frac{h}{h} \qquad (9.8)$$

h and h' will be taken positive or negative in accordance with the accepted sign convention. In triangles A′B′P and ABP, we have,

$$\frac{B\ A}{BA} \qquad \frac{B\ P}{BP}$$

With the sign convention, this becomes

$$\frac{-h}{h} \qquad \frac{-v}{-u}$$

so that

$$m = \frac{h}{h} \qquad -\frac{v}{u} \qquad (9.9)$$

We have derived here the mirror equation, Eq. (9.7), and the magnification formula, Eq. (9.9), for the case of real, inverted image formed by a concave mirror. With the proper use of sign convention, these are, in fact, valid for all the cases of reflection by a spherical mirror (concave or convex) whether the image formed is real or virtual. Figure 9.6 shows the ray diagrams for virtual image formed by a concave and convex mirror. You should verify that Eqs. (9.7) and (9.9) are valid for these cases as well.

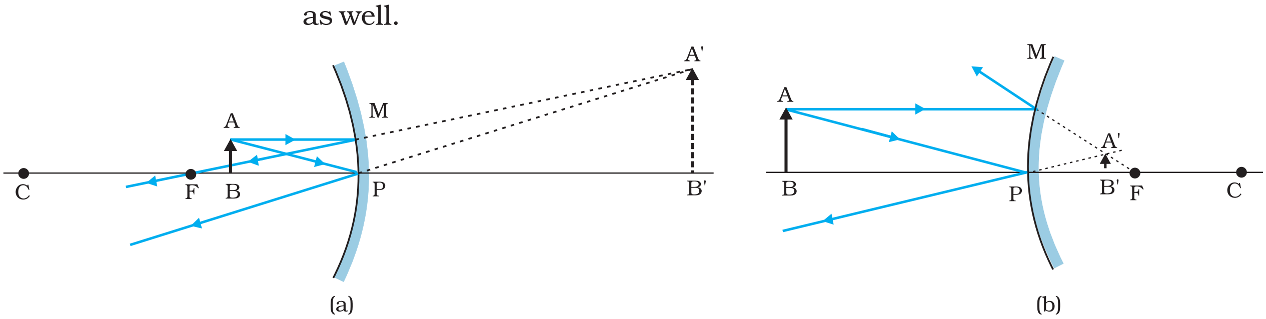

FIGURE 9.6 Image formation by (a) a concave mirror with object between P and F, and (b) a convex mirror.

Example 9.1 Suppose that the lower half of the concave mirror's reflecting surface in Fig. 9.5 is covered with an opaque (non-reflective) material. What effect will this have on the image of an object placed in front of the mirror?

Solution You may think that the image will now show only half of the object, but taking the laws of reflection to be true for all points of the remaining part of the mirror, the image will be that of the whole object. However, as the area of the reflecting surface has been reduced, the intensity of the image will be low (in this case, half).

EXAMPLE 9.1

Example 9.2 A mobile phone lies along the principal axis of a concave mirror, as shown in Fig. 9.7. Show by suitable diagram, the formation of its image. Explain why the magnification is not uniform. Will the distortion of image depend on the location of the phone with respect to the mirror?

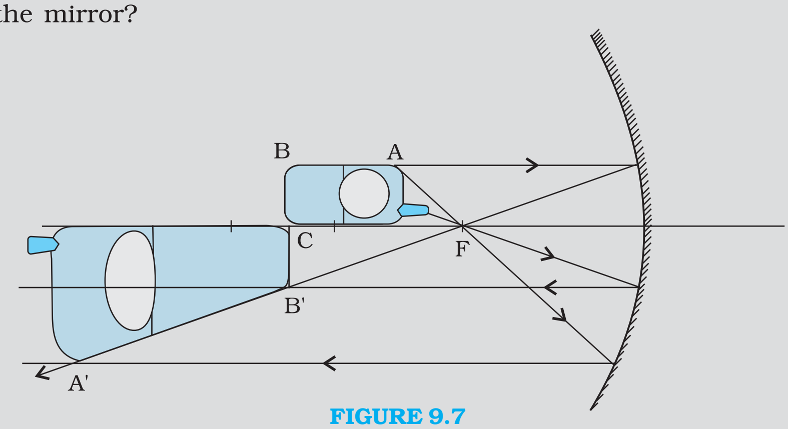

FIGURE 9.7

Solution

The ray diagram for the formation of the image of the phone is shown in Fig. 9.7. The image of the part which is on the plane perpendicular to principal axis will be on the same plane. It will be of the same size, i.e., B′C = BC. You can yourself realise why the image is distorted.

EXAMPLE 9.2

Example 9.3 An object is placed at (i) 10 cm, (ii) 5 cm in front of a concave mirror of radius of curvature 15 cm. Find the position, nature, and magnification of the image in each case.

Solution

The focal length $f = -15/2$ cm $= -7.5$ cm

(i) The object distance $u = -10$ cm. Then Eq. (9.7) gives

$$\frac{1}{v} \quad \frac{1}{-10} \quad \frac{1}{-7.5}$$

or $\quad v = \frac{10 \quad 7.5}{2.5} = -\,30$ cm

The image is 30 cm from the mirror on the same side as the object.

Also, magnification $m = -\frac{v}{u} \quad -\frac{(\ 30)}{(\ 10)} \quad -3$

The image is magnified, real and inverted.

EXAMPLE 9.3

EXAMPLE 9.3

(ii) The object distance $u = -5$ cm. Then from Eq. (9.7),

$$\frac{1}{v} \quad \frac{1}{5} \quad \frac{1}{7.5}$$

or $$v = \frac{5 \quad 7.5}{7.5-5} \quad 15 \text{ cm}$$

This image is formed at 15 cm behind the mirror. It is a virtual image.

Magnification $m = -\frac{v}{u} \quad -\frac{15}{(\ 5)} \quad 3$

The image is magnified, virtual and erect.

EXAMPLE 9.4

Example 9.4 Suppose while sitting in a parked car, you notice a jogger approaching towards you in the side view mirror of $R = 2$ m. If the jogger is running at a speed of 5 m s^{-1}, how fast the image of the jogger appear to move when the jogger is (a) 39 m, (b) 29 m, (c) 19 m, and (d) 9 m away.

Solution

From the mirror equation, Eq. (9.7), we get

$$v \quad \frac{fu}{u \quad f}$$

For convex mirror, since $R = 2$ m, $f = 1$ m. Then

for $u = -39$ m, $v \quad \frac{(\ 39) \quad 1}{39 \quad 1} \quad \frac{39}{40}$ m

Since the jogger moves at a constant speed of 5 m s^{-1}, after 1 s the position of the image v (for $u = -39 + 5 = -34$) is (34/35)m.

The shift in the position of image in 1 s is

$$\frac{39}{40} \quad \frac{34}{35} \quad \frac{1365 \quad 1360}{1400} \quad \frac{5}{1400} \quad \frac{1}{280} \text{m}$$

Therefore, the average speed of the image when the jogger is between 39 m and 34 m from the mirror, is (1/280) m s^{-1}

Similarly, it can be seen that for $u = -29$ m, -19 m and -9 m, the speed with which the image appears to move is

$$\frac{1}{150}\text{m s}^{-1}, \ \frac{1}{60}\text{m s}^{-1} \text{ and } \frac{1}{10}\text{m s}^{-1}, \text{ respectively.}$$

Although the jogger has been moving with a constant speed, the speed of his/her image appears to increase substantially as he/she moves closer to the mirror. This phenomenon can be noticed by any person sitting in a stationary car or a bus. In case of moving vehicles, a similar phenomenon could be observed if the vehicle in the rear is moving closer with a constant speed.

9.3 REFRACTION

When a beam of light encounters another transparent medium, a part of light gets reflected back into the first medium while the rest enters the other. A ray of light represents a beam. The direction of propagation of an obliquely incident ray of light that enters the other medium, changes