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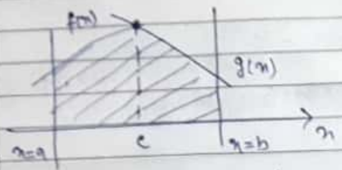
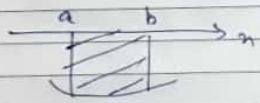
Area

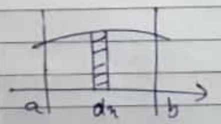


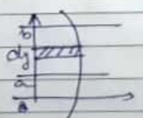
Area : $\int_a^c f(x)\,dx + \int_c^b g(x)\,dx$

Only if f(x) and g(x) are above x axis.
If ~~graph is below~~



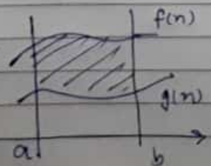
Area : $-\int_a^b f(x)\,dx$





$\int_a^b y\,dx$

$\int_a^b x\,dy$



$\int_a^b (f(x) - g(x))\,dx$

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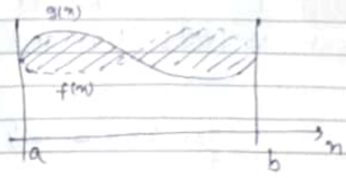
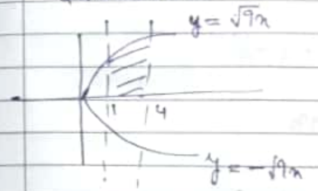


$\int_a^c g(x) - f(x) + \int_c^b f(x) - g(x)$

Q find the area bounded by the curve $y^2 = 9x$ & lines $x=1$, $x=4$ and the x axis in first Quadrant.

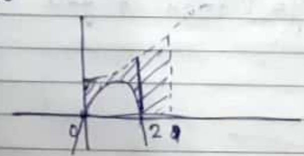


$\int_1^4 \sqrt{9x}\,dx$

$3 \dfrac{x^{3/2}}{3/2}\Big|_1^4$

$16 - 2 = \boxed{14}$

Q find area of region bounded by $y = 2^x$, $y = 2x - x^2$, ordinates $x=0$, $x=2$.



$\int_0^2 2^x\,dx$

$\dfrac{2^x}{\ln 2}\Big|_0^2$

$-(x)(x-2)$

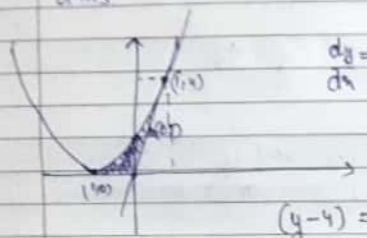
$\dfrac{4-1}{2\ln 2} - \int_0^2 2x - x^2$

$\boxed{\dfrac{3}{\ln 2} - \dfrac{4}{3}}$

$\dfrac{15}{\ln 2} - \left(x^2 - \dfrac{x^3}{3}\right)\Big|_0$

Scanned with CamScanner

Q Area bounded by $y = x^2+2x+1$, the tangent at ~~(1,4)~~ (1,4) and the y axis



$\frac{dy}{dx} = 2x+2 = 4$

$(x+1)^2 = y$

$(y-4) = 4(x-1)$

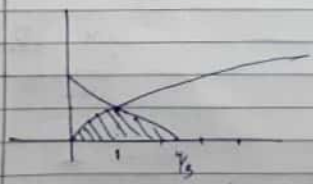
$y = 4x - \cancel{4} + \cancel{4}$

$\int_0^1 x^2+2x+1 - 4x$

$\frac{(x-1)^3}{3}\Big|_0^1 = \boxed{\frac{1}{3}}$

Q find the area bounded by

$y = \sqrt{x}$, $y = \sqrt{4-3x}$, $y=0$.

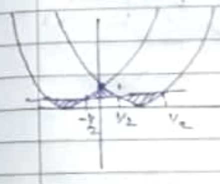


$\int_0^1 \sqrt{x}\,dx + \int_1^{4/3} \sqrt{4-3x}$

$\frac{x^{3/2}}{3/2}\Big|_0^1 + \frac{(4-3x)^{3/2}}{-3\times 3/2}\Big|_1^{4/3}$

$\frac{3\times 2}{3\times 3} + \frac{2}{9} = \boxed{\frac{8}{9}}$

Q find the area of region bounded by $y = (x+1)^2$, $y = (x-1)^2$ & $y = 1/4$.



$\frac{1}{2}$

$\int_0^{1/2} (x-1)^2$

$2\times\left[\frac{(x-1)^3}{3}\Big|_0^{1/2}\right] - \frac{1}{4}$

$2\times\left[\frac{1}{24} + \frac{8}{8\times 3}\right.$ $\frac{9}{24}$ $\frac{9}{12} - \frac{3}{4\times 3} = \frac{6}{12} = \boxed{\frac{1}{2}}$

$\frac{(x-1)^3}{3}\Big|_{1/2}^{3/2}$

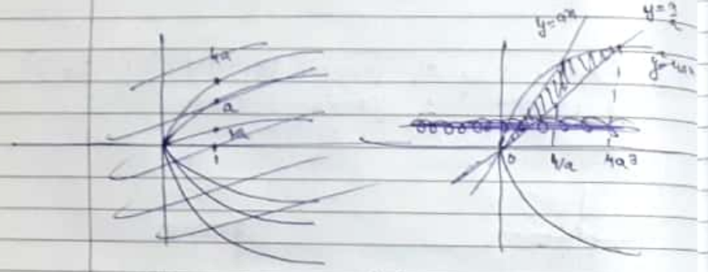
$+$

$\frac{1}{8\times 3} + \frac{1}{8\times 3} = \frac{1}{12} + \frac{1}{12} = \frac{1}{6} + \frac{3}{2\times 3} = \boxed{\frac{2}{3}}$

Q find the max area bounded by the curves

$y^2 = 4ax$, $y = ax$, $y = \frac{x}{a}$

a belongs $[1, 2]$



$$\left(\int_0^{4/a} ax - \frac{x}{a}\right) + \int_{4/a}^{4a^3} \left(\sqrt{4ax} - \frac{x}{a}\right) dx$$

$$ax^2 - \frac{x^2}{a}\Big|_0^{4/a} + \left(\frac{(4ax)^{3/2}}{3/2 \cdot 4a}\right) - \frac{x^2}{a}\Big|_{4/a}^{4a^3}$$

$$\frac{16}{a} - \cancel{\frac{16}{a^3}} + \frac{32 \cdot 64 a^6}{3 \cdot 4a} - 16a^5 - \frac{16 \times 2}{3} + \cancel{\frac{16}{a^3}}$$

$$\frac{16}{a} - \frac{80a^5}{3} - \frac{32}{3} + \cancel{\frac{32}{3}a^3}$$

$$\frac{-16}{a^2} - 80a^4 \qquad a^6 = \frac{16}{80} \qquad 16$$

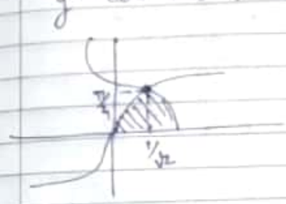
$$-\frac{(16 - 80a^6)}{a^2} = 0 \qquad \frac{48 - 32}{3}$$

$$\boxed{\frac{16}{3}}$$

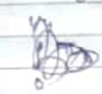
Q find the area bounded by $y = \sin^{-1}(x)$

$y = \cos^{-1}(x)$, x axis.

$$\frac{1}{2}\int_0^{1/\sqrt{2}} \frac{dx}{\sqrt{1-x^2}}$$



$$\int_0^{1/\sqrt{2}} \sin^{-1}(x) + \int_{1/\sqrt{2}}^{1} \cos^{-1}(x).$$



$$\int_0^{1/\sqrt{2}} \sin^{-1}(x) + \int_{1/\sqrt{2}}^{1} \cos^{-1}(x). = (\sqrt{2} - 1)$$

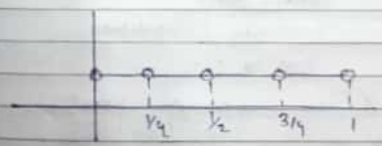
$$\int_0^{\pi/4} \cos(y)\, dy - \int_0^{\pi/4} \sin(y)\, dy. = (\sqrt{2} - 1)$$

CM Test 2 Doubt

$$f(x) = \frac{\{x\} g(x)}{\{x\} g(x)}$$

Periodic $\frac{1}{4}$

$g(x)$ is diff



$\Rightarrow g(x) = 0$ at $x = I + \frac{1}{4}$

Some part of Area in physics notes

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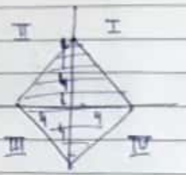
Q $|x| + |y| = 4$ find the area



$x+y=4$ I
$-x+y=4$ II
$-x-y=4$ III
$x-y=4$ IV

$(4\sqrt{2})^2 = \boxed{32}$

using concept of coordinate geometry

shifting of origin

$f(x,y) \to f(x-a, y-b)$ new axes

Note that this type of eqn are symmetric in all 4 quadrants as no change after putting $\pm x$ or $\pm y$.

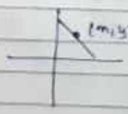
$|x-1| + |y+2| = 4$
$|X| + |Y| = 4$

$X = x-1$ new axes
$Y = y+2$ new axes

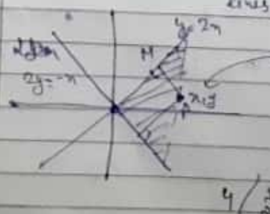
No change in ~~the~~ Ans just shifting of origin $(1,-2)$

Q find the area $|2x-y| + |x+2y| = 4$

$|x| + |y| = 4$ shifting of Axis



Locus of point whose sum of ⊥ distance from mutually perpendicular lines ~~of~~ is 4.



$\left|\frac{2x-y}{\sqrt{5}}\right| + \left|\frac{x+2y}{\sqrt{5}}\right| = \ldots$ Normal form

$|PM| + |PN| = \frac{4}{\sqrt{5}}$

$4\left(\frac{1}{2} \times \frac{4}{\sqrt{5}} \times \frac{4}{\sqrt{5}}\right) = \frac{32}{5}$

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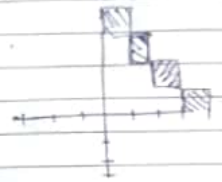
Q $[|x|] + [|y|] = 3.$



$y>0 \quad x>0$

$[x] + [y] = 3$

$1 > y > 0$
$3 < x < 4$

$2 > y > 1$
$3 > y > 2$ $2 < x < 3$
$4 > y > 3$ $1 < x < 2$
$0 < x < 1$

$[x]+[y]=$	
0	3
1	2
2	1
3	0

graph is symmetric.

$(4) \times 4 = \boxed{16}$

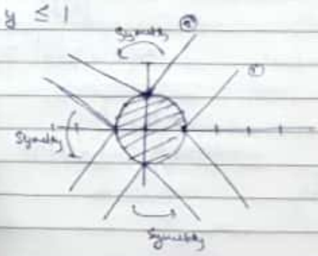
Q $\big||x| - |y|\big| \leq 1 \qquad x^2 + y^2 \leq 1$

graph is symmetric about axises.

$x \geq 0 \quad y \geq 0$

$-1 \leq x - y \leq 1$

$\boxed{\pi}$

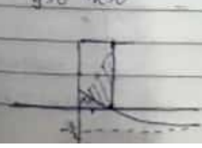


Q $|y| \geq e^{-|x|} - \frac{1}{2}$ and $\frac{|x|+|y|}{2} + \left|\frac{|x|-|y|}{2}\right| \leq 2$

$y>0 \quad x>0$

$\frac{x+y}{2} + \left|\frac{x-y}{2}\right| \leq 2$

$x > y$: $x \leq 2$
$x < y$: $y \leq 2$



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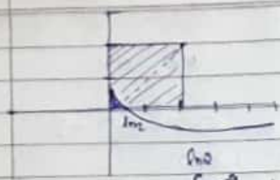


$e^{-x} = 1/2$

$-x = \ln 1/2$

$x = \ln 2$

$$\int_0^{\ln 2} e^{-x} - \frac{1}{2}$$

$$\frac{e^{-x}}{-1} - \frac{1}{2}x \Big|_0^{\ln 2}$$

$$\left[4 - \left(+1 - \frac{1}{2} - \frac{1}{2}\ln 2\right)\right] \Rightarrow \left[\frac{7}{2} + \frac{1}{2}\ln 2\right] \times 4$$

Symmetry across 4 quad

Ans $\boxed{14 + \ln 4}$

Q Show that $\int_a^b f(x)\,dx + \int_{f(a)}^{f(b)} f^{-1}(x)\,dx$

$= bf(b) - af(a)$

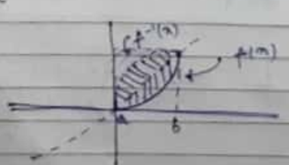
$f^{-1}(x) = t$

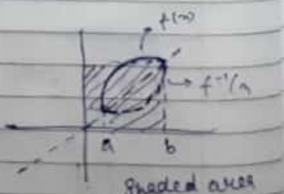
$x = f(t)$

$dx = f'(t)\,dt$

$$[t f(t)]_a^b - \int_a^b f(t)\,dt.$$

Mathematical proof & Key with graph





Shaded area

$bf(b) - af(a)$

bigger

smaller

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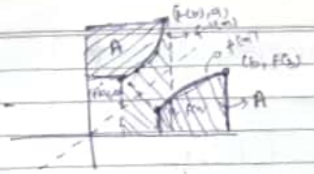
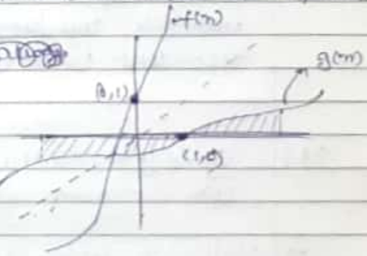


Q Let $f(x) = x^3 - 2x^2 + 4x + 1$ and $g(x)$ be its inverse then find the area of region bounded by $g(x)$, x axis and ordinates $x = -23$, $x = 4$



$$\int_1^4 g(x) - \int_{-23}^1 g(x)$$

Method ①

$$\left[\int_0^1 f(x) + \int_1^4 g(x)\right] - \left[\int_{-23}^1 g(x) + \int_{-2}^0 f(x)\right] - \int_0^1 f(x) + \int_{-2}^0 f$$

$$\left[(4-1) - (1 - 46)\right] - \int \frac{x^4}{4} - \frac{2x^3}{3} + 2x^2 + x \Big|_0^1$$

$$(3 + 45) - \left[\frac{1}{4} - \frac{2}{3} + 2 + 1\right]$$

$12 + 27 - 18$

$8 - 8 - 8 +$

$48 - \frac{1}{4} + \frac{2}{3} - 3$

$$+ \frac{x^4}{4} - \frac{2x^3}{3} + 2x^2 + x \Big|_{-2}^0$$

$$- \left[4 + \frac{16}{3} + 8 - 2\right]$$

Ans $\boxed{385}$

$= \left(\frac{12 \times 35 - 18 \cdot 3 + 2}{...}\right)$ $45 - 10$