

Definite Integration

$$\int_a^b f(x) dx = (g(x))_a^b = g(b) - g(a)$$

ii) continuous on (a, b)

iii) $\left. \begin{array}{l} g(b^-) - g(a) \\ g(b) - g(a^+) \\ g(b^-) - g(a^+) \end{array} \right\}$ if $f(x)$ is not defined

eg:- $\int_0^1 \ln x dx = (x \ln x - x)_0^1 = -1 - \lim_{x \rightarrow 0^+} x \ln x$
 $= -1 - 0 = -1$

Q $\int_{\frac{1}{2}}^2 (1+x - \frac{1}{x}) e^{x+\frac{1}{x}} dx$

Ans $\int_{\frac{1}{2}}^2 \underbrace{\left(e^{x+\frac{1}{x}} + x f'(x) \right)}_{f(x)} dx$

$$= \left(x e^{x+\frac{1}{x}} \right)_{\frac{1}{2}}^2$$

Q $\int_3^5 \frac{x^2 dx}{\sqrt{(x-3)(5-x)}}$

Ans $x = 3 \cos^2 \theta + 5 \sin^2 \theta$

$$2 \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} (9 + 4 \sin^4 \theta + 12 \sin^2 \theta) d\theta \Rightarrow \text{we get } \frac{33\pi}{2}$$

Properties

①
$$\int_a^b f(x) dx = \int_a^b f(t) dt$$

②
$$\int_b^a f(x) dx = - \int_a^b f(x) dx$$

③
$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

continuous in $a \rightarrow c$
continuous in $c \rightarrow b$

c may not necessarily be in between a to b as $-\int_a^b f(x) = +\int_b^a f(x)$

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$$\int_a^b f(x) dx = \int_{a+c}^{b+c} f(x-c) dx \quad : c > 0$$

*
$$k \int_{\frac{a}{k}}^{\frac{b}{k}} f(kx) dx = \int_a^b f(x) dx$$

eg:-
$$\int_0^{\pi/2} f(\sin^2 x) dx = \frac{1}{2} \int_0^{\pi/2} f(\sin x) dx$$

here $k = \frac{1}{2}$

$$\int_{\frac{1}{2}}^1 \frac{1}{x} \operatorname{cosec} \left(x - \frac{1}{x} \right) dx$$

$x \rightarrow \frac{1}{t}$

$$\int_{\frac{1}{2}}^1 \frac{1}{x} \operatorname{cosec} \left(x - \frac{1}{x} \right) dx = \int_{\frac{1}{2}}^1 \frac{1}{\frac{1}{t}} \operatorname{cosec} \left(\frac{1}{t} - t \right) \left(-\frac{1}{t^2} \right) dt$$

$$= - \int_{\frac{1}{2}}^2 \frac{1}{x} \operatorname{cosec}^{-1} \left(x - \frac{1}{x} \right)$$

$$I = -I$$

↓

$$\boxed{I=0}$$

Q $\int_0^{\infty} \frac{\ln x}{1+x^2} dx$

Ans. $x \rightarrow \frac{1}{t} \int_0^{\infty} \frac{(-\ln t)t^2}{1+t^2} \times \frac{(-dt)}{t^2} = - \int_0^{\infty} \frac{\ln x}{1+x^2} dx$

$$I = -I$$

$$I = 0$$

Q $\int_0^{\infty} \frac{\tan^{-1} x}{x^2 - x + 1} dx$

Ans. $x = \frac{1}{t} \quad dx = \frac{-1}{t^2} dt$

$$I = \int_0^{\infty} \frac{-\cot^{-1}(t)}{t^2 - t + 1} dt$$

$$I + I = \int_0^{\infty} \frac{\pi}{2(t^2 - t + 1)}$$

$$Q \int_0^{\infty} \frac{dx}{(1+x^2)(1+x^{17})}$$

$$x = \frac{1}{t}$$

$$I = \int_0^{\infty} \frac{x^{17} dx}{(1+x^2)(1+x^{17})}$$

$$2I = \int_0^{\infty} \frac{dx}{1+x^2}$$

$$2I = \frac{\pi}{2} \quad I = \frac{\pi}{4}$$

By Parts in Definite Integration

$$\int_a^b v \cdot u dx = u \int_a^b v dx - \int_a^b (u' \int_a^b v dx) dx$$

+ ya fir whole pe hi limit lga de!

$$Q \int_0^{\frac{\pi}{2}} \frac{(\cos x)^{\sqrt{2}+1} dx}{(\cos x)^{\sqrt{2}-1} dx}$$

$$Ans \int_0^{\frac{\pi}{2}} (\cos x)^{\sqrt{2}} (\cos x) dx = \left((\cos x)^{\sqrt{2}} (\sin x) \right)_0^{\frac{\pi}{2}} + \sqrt{2} \int_0^{\frac{\pi}{2}} (\cos x)^{\sqrt{2}-1} (\sin^2 x) dx$$

f(x) =

$$\int_0^{\frac{\pi}{2}} (\cos x)^{\sqrt{2}+1} = \sqrt{2} \int_0^{\frac{\pi}{2}} (\cos x)^{\sqrt{2}-1} - \sqrt{2} \int_0^{\frac{\pi}{2}} (\cos x)^{\sqrt{2}+1}$$

Ans \Rightarrow $\frac{\sqrt{2}}{\sqrt{2}+1}$

$$\phi \int_0^1 \frac{(1 - (1-x^3)^{\sqrt{2}})^{\sqrt{3}} x^2 dx}{(1 - (1-x^3)^{\sqrt{2}})^{\sqrt{3}+1} x^2 dx}$$

$$1-x^3 = t$$

$$= \int_0^1 \frac{(1-t^{\sqrt{2}})^{\sqrt{3}} dt}{(1-t^{\sqrt{2}})^{\sqrt{3}+1} dt} \rightarrow I_1$$

$$\int_0^1 \frac{(1-t^{\sqrt{2}})^{\sqrt{3}} dt}{(1-t^{\sqrt{2}})^{\sqrt{3}+1} dt} \rightarrow I_2$$

$$I_2 = \int_0^1 \frac{1 \cdot (1-t^{\sqrt{2}})^{\sqrt{3}+1} dt}{(1-t^{\sqrt{2}})^{\sqrt{3}+1} dt}$$

$$= \left((1-t^{\sqrt{2}})^{\sqrt{3}+1} t \right)_0^1 + \int_0^1 \frac{(\sqrt{3}+1)(1-t^{\sqrt{2}})^{\sqrt{3}}}{\sqrt{2}(t^{\sqrt{2}-1})t} dt$$

$$\int_0^1 (\sqrt{3}+1)\sqrt{2} (1-t^{\sqrt{2}})^{\sqrt{3}} (1-t^{\sqrt{2}-1}) dt$$

④ King

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$

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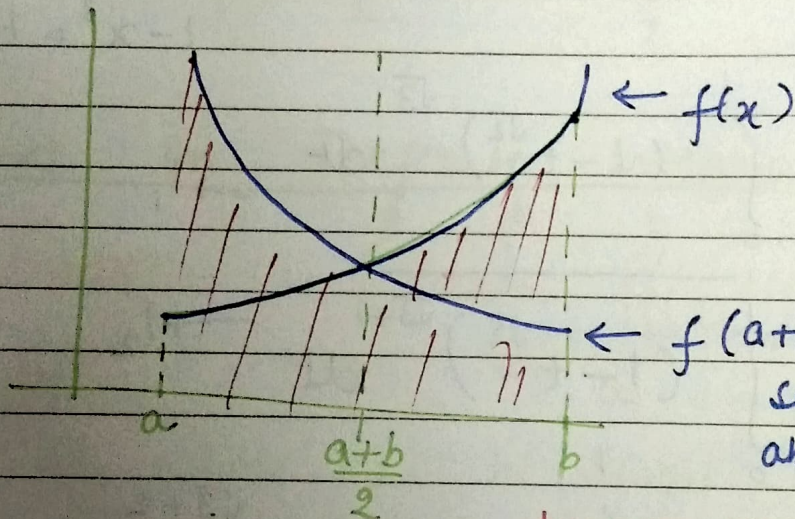
odd = 0

even = even

⑤ Queen

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

Proof



hence proved

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} dx}{\sqrt{\sin x} + \sqrt{\cos x}}$$

King

$$1 = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x} dx}{\sqrt{\sin x} + \sqrt{\cos x}}$$

$$2 \cdot 1 = \int_0^{\frac{\pi}{2}} 1 dx \Rightarrow 1 = \frac{\pi}{4}$$

$$Q \quad I = \int_{50}^{100} \frac{\ln x}{\ln x + \ln(150-x)} dx$$

Ans

$$I = \int_{50}^{100} \frac{\ln(150-x)}{\ln(150-x) + \ln x} dx$$

$$2I = \int_{50}^{100} dx \quad I = 25$$

$$Q \quad \int_0^{\pi} \frac{2x \sin x}{3 + \cos 2x} dx$$

$$I = \int_0^{\pi} \frac{2(\pi-x) \sin x}{3 + \cos 2x} dx$$

$$2I = 2 \int_0^{\pi} \frac{\cancel{\sin x} \pi \sin x}{3 + \cos 2x} dx$$

we get $\frac{\pi^2}{4}$

$$Q \quad \int_0^2 \frac{dx}{(17+8x-4x^2)(e^{(1-x)}+1)}$$

applying

$$I = \int_0^2 \frac{e^{(1-x)} dx}{(17+8x-4x^2)(e^{6(1-x)}+1)}$$

$$2I = \int_0^2 \frac{dx}{\text{Quad.}} \Rightarrow I = \frac{1}{2\sqrt{2}} \ln\left(\frac{\sqrt{21}-2}{\sqrt{21}+2}\right)$$

$$\int_0^1 \cot^{-1}(1+x^2-x) dx$$

Ans. $\int_0^1 \tan^{-1} \left(\frac{x+(1-x)}{1+x(x-x)} \right) dx$

$$I = \int_0^1 (\tan^{-1} x + \tan^{-1}(1-x)) dx$$

{ indefinite main $x-1, 1-x \dots > < 0$
Sochna nhi hai, definite hain hai }

$$2I = 2 \int_0^1 \tan^{-1} x dx$$

$$I = \int_0^1 \tan^{-1} x dx$$

$$\Rightarrow \left(\frac{\pi}{4} - \ln 2 \right)$$

Q $\int_0^4 x(x-1)(x-2)(x-3)(x-4) dx$

Ans. $2I = \int_0^4 0$

$$I = 0$$

Q $\int_0^1 \frac{\ln(1+x)}{1+x^2} dx$

Ans. $\int_0^{\pi/4} \ln(1+\tan \theta) d\theta \quad x \rightarrow \tan \theta$

$$= \int_0^{\pi/4} \ln(\sin \theta + \cos \theta) d\theta - \int_0^{\pi/4} \ln(\cos \theta) d\theta$$

P-6.

$$\int_{-a}^a f(x) dx = \int_0^a (f(x) + f(-x)) dx = \begin{cases} 2 \int_0^a f(x) dx : \text{even} \\ 0 : \text{odd} \end{cases}$$

Q $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\sec x}{|x|} \ln\left(\frac{1-x}{1+x}\right) dx$

= 0

Q $\int_{-1}^3 \tan^{-1}\left(\frac{x}{x^2+1}\right) + \tan^{-1}\left(\frac{x^2+1}{x}\right) dx$

Ans. $\int_{-1}^1 () dx + \int_1^3 \left(\tan^{-1}\left(\frac{x}{x^2+1}\right) + \cot^{-1}\left(\frac{x}{x^2+1}\right) \right) dx$

= $\left[\frac{\pi}{2} x \right]_1^3 \Rightarrow \underline{\underline{\pi}}$

Q $\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \frac{x^4}{1-x^4} \cos^{-1}\left(\frac{2x}{1+x^2}\right) dx$

Ans. $I = \int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \frac{x^4}{1-x^4} \cos^{-1}\left(\frac{-2x}{1+x^2}\right) dx = \int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \frac{x^4 \pi}{1-x^4} - \frac{x^4 \cos^{-1}\left(\frac{2x}{1+x^2}\right)}{1-x^4} dx$

King Queen

= $\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \frac{x^4 \pi}{1-x^4} = \frac{\pi^2}{12} - \frac{\pi}{3} - \frac{\pi}{4} \ln(2-\sqrt{3})$

$$Q \int_{-1}^1 \frac{dx}{(x^2+x+1) + \sqrt{x^4+3x^2+1}}$$

$$= \int_{-1}^1 \frac{(x^2+x+1) - \sqrt{x^4+3x^2+1}}{2x(x^2+1)}$$

$$= \int_{-1}^1 \frac{(x^2+x+1)dx}{2x(x^2+1)} = \int_{-1}^1 \frac{dx}{2x} + \int_{-1}^1 \frac{dx}{2(x^2+1)}$$

Q P-7

$$\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a-x) dx$$

(use as part)

Also, $\int_0^{2a} f(x) dx = \begin{cases} 2 \int_0^a f(x) dx : f(2a-x) = f(x) \\ 0 : f(2a-x) = -f(x) \end{cases}$

shifted \rightarrow $f(x)$ is even to $x=a$ symm. about a

shifted odd to $x=a$

eg:- $\int_0^{2\pi} \cos^7 x dx = 2 \int_0^{\pi} \cos^7 x dx = 0$
 $= 2 \int_0^{\frac{\pi}{2}} \cos^7 x dx = 0$
 odd at $\frac{\pi}{2}$

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